A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 4: BINOMIAL EXPANSION · A* EXAM PRACTICE

Hints online, answers free with an account:
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Binomial expansion: A* questions

80 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 Lena wants to solve the equation $\sqrt{1+x}+\sqrt{1-2x}=1.98$.

(a) Find the binomial expansions of $\sqrt{1+x}$ and of $\sqrt{1-2x}$, in ascending powers of x, up to and including the term in x^2. [3 marks]

Answer .....

(b) Hence show that, for small values of x, the equation becomes $5x^2+4x-0.16=0$. [3 marks]

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(c) Solve $5x^2 + 4x - 0.16 = 0$ to find an approximate solution of Lena's equation, giving your answer to 3 significant figures. Explain why the other root of the quadratic is not a solution. [3 marks]

Answer $x \approx \ldots$

Total for X1: 9 marks

X2 $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$

(a) Find the binomial expansion of $f(x)$ in ascending powers of x up to the term in x^6. [3 marks]

Answer .....

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(b) Given that $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \frac{\pi}{6}$, use your answer to (a) to find an approximation for π. Give your answer to 4 decimal places. [6 marks]

Answer $\pi \approx \ldots$

(c) Explain whether your answer to (b) is an overestimate or an underestimate of π. [2 marks]

Total for X2: 11 marks

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X3 Find the coefficients of x^2 and of x^3 in the expansion of $\dfrac{(2+x)^5}{\sqrt{1-2x}}$ [9 marks]

in ascending powers of x. State the values of x for which the expansion is valid.

Answer x^2 : ... , x^3 : ... , valid for ...

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X4 $g(x) = \dfrac{8 - 7x}{(1 - 2x)(2 - x)}$

(a) Express $g(x)$ in partial fractions. [3 marks]

Answer .....

(b) Show that the coefficient of x^r in the expansion of $g(x)$ is $3 \times 2^r + \left(\dfrac{1}{2}\right)^r$. [4 marks]

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(c) Find the smallest value of r for which the coefficient of x^r is greater than $10\,000$. [3 marks]

Answer $r = \ldots$

(d) State the values of x for which the expansion of $g(x)$ is valid. [2 marks]

Answer ...

Total for X4: 12 marks

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X5 In the theory of relativity, a particle of mass m moving at speed v has kinetic energy $E = (\gamma - 1)mc^2$, where $\gamma = \left(1 - \dfrac{v^2}{c^2}\right)^{-\frac{1}{2}}$ and c is the speed of light.

(a) Find the binomial expansion of γ in ascending powers of $\dfrac{v}{c}$, up to the term in $\dfrac{v^4}{c^4}$. [3 marks]

Answer .....

(b) Hence show that $E \approx \dfrac{1}{2}mv^2 + \dfrac{3mv^4}{8c^2}$. [2 marks]

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(c) The everyday formula is $E = \frac{1}{2}mv^2$. Using the exact formula for E, find the percentage error in the everyday formula when $v = 0.1c$. Give your answer to 2 significant figures. [3 marks]

Answer ... %

(d) State the values of v for which the expansion in (a) is valid, and give one reason why the everyday formula is still used for a car. [2 marks]

Total for X5: 10 marks

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X6 For small values of x, $\sqrt[3]{8+6x} \approx \dfrac{16+ax}{8+bx}$, where a and b are constants. The binomial expansions of the two sides, in ascending powers of x, agree up to and including the term in x^2. Find the values of a and b. Hence, by choosing a suitable value of x, find an approximation for $\sqrt[3]{9}$, giving your answer as a fraction in its simplest form. [8 marks]

Answer $a = \ldots$, $b = \ldots$, $\sqrt[3]{9} \approx \ldots$

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X7 $h(x) = \dfrac{3x^2 + 8x + 13}{(1+x)^2(3-x)}$

(a) Express $h(x)$ in the form $\dfrac{A}{1+x} + \dfrac{B}{(1+x)^2} + \dfrac{C}{3-x}$. [4 marks]

Answer $A = \ldots$, $B = \ldots$, $C = \ldots$

(b) Hence find the expansion of $h(x)$ in ascending powers of x up to the term in x^2. Give each coefficient as an exact fraction. [5 marks]

Answer .....

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(c) State, with a reason, the values of x for which this expansion is valid. [2 marks]

Answer ...

Total for X7: 11 marks

X8 $k(x) = \dfrac{1}{\sqrt{1+4x}}$

(a) Find the binomial expansion of $k(x)$ in ascending powers of x up to the term in x^3. [4 marks]

Answer .....

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(b) By substituting $x = -\frac{1}{200}$ into your expansion, find an approximation for $\sqrt{2}$. Give your answer to 5 decimal places. [4 marks]

Answer $\sqrt{2} \approx \ldots$

(c) Jo says: "I could find $\frac{1}{\sqrt{3}}$ the same way by putting $x = \frac{1}{2}$." Explain why Jo is wrong. [2 marks]

Total for X8: 10 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find unknown constants from the first terms of an expansion.	○	○	○
I can take out the constant before expanding and state where the expansion is valid.	○	○	○
I can use partial fractions to find a general term.	○	○	○
I can use an expansion to approximate a value and judge the error.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Use $(1+u)^{\frac{1}{2}} = 1 + \frac{1}{2}u - \frac{1}{8}u^2 + \ldots$, first with $u = x$, then with $u = -2x$.

Hint 2 Add the two expansions, set the sum equal to 1.98, then multiply through by 8.

Hint 3 Each expansion is valid only for some values of x. Check each root against $|2x| < 1$.

X2

Hint 1 Use the expansion of $(1+u)^{-\frac{1}{2}}$ with $u = -x^2$.

Hint 2 Integrate each term of your series from 0 to $\frac{1}{2}$, then multiply by 6.

Hint 3 Look at the signs of the coefficients of the terms that you did not use.

X3

Hint 1 $(2+x)^5$ is a finite expansion, so its validity is never a problem.

Hint 2 Expand $(1-2x)^{-\frac{1}{2}}$ up to x^3 and $(2+x)^5$ up to x^3.

Hint 3 For the x^3 term you need four products: x^0x^3, x^1x^2, x^2x^1 and x^3x^0.

X4

Hint 1 Put $x = \frac{1}{2}$ and $x = 2$ into $8 - 7x \equiv A(2-x) + B(1-2x)$.

Hint 2 Take the 2 out of $2 - x$: $\frac{2}{2-x} = \frac{1}{1-\frac{x}{2}}$. Each fraction is a geometric series.

Hint 3 The expansion is valid only where both expansions are valid: take the smaller range.

X5

Hint 1 Treat $\frac{v^2}{c^2}$ as one quantity u and expand $(1-u)^{-\frac{1}{2}}$.

Hint 2 For (c) put $\frac{v}{c} = 0.1$, so $\frac{v^2}{c^2} = 0.01$, and work out $\gamma - 1$ exactly.

Hint 3 Percentage error $= \frac{\text{exact} - \text{approximate}}{\text{exact}} \times 100$.

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X6

Hint 1 Write $\dfrac{16+ax}{8+bx} = 2\left(1+\dfrac{a}{16}x\right)\left(1+\dfrac{b}{8}x\right)^{-1}$ and expand each side up to x^2.

Hint 2 Equate the coefficients of x and of x^2. The first gives $a-2b=4$; put $2b-a=-4$ into the second.

Hint 3 You need $8+6x=9$, and that x must lie inside $|x|<\dfrac{4}{3}$.

X7

Hint 1 Put $x=-1$ and $x=3$ into the identity, then compare the x^2 terms to find A.

Hint 2 For $\dfrac{4}{3-x}$, take the 3 out first: $\dfrac{4}{3-x}=\dfrac{4}{3}\left(1-\dfrac{x}{3}\right)^{-1}$.

Hint 3 Each part has its own range of validity. The whole expansion needs all of them.

X8

Hint 1 Use $(1+u)^{-\frac{1}{2}}$ with $u=4x$.

Hint 2 With $x=-\dfrac{1}{200}$, $1+4x=\dfrac{49}{50}$, and $\sqrt{\dfrac{50}{49}}=\dfrac{\sqrt{50}}{7}=\dfrac{5\sqrt{2}}{7}$.

Hint 3 Find the range of x for which the expansion is valid.

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