A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 6: TRIGONOMETRIC FUNCTIONS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Trigonometric functions: A* questions

73 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 Solve, for $0 \leq \theta < \pi$, the equation [9 marks]

$$2\tan^2\left(2\theta - \frac{\pi}{3}\right) + \sec\left(2\theta - \frac{\pi}{3}\right) = 1.$$

Give exact answers where possible, and other answers to 3 significant figures.

Answer $\theta = \ldots$

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X2 The function f is defined by $f(x) = 3 + 2\,\text{cosec}\,x$, $0 < x \leq \frac{\pi}{2}$.

(a) Find the range of f. [2 marks]

Answer .....

(b) Find $f^{-1}(x)$, stating its domain, and find the exact value of $f^{-1}(7)$. [4 marks]

Answer .....

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(c) Solve $f(x) = 3 + \cot^2 x$, giving your answer to 3 significant figures. [4 marks]

Answer $x = \ldots$

Total for X2: 10 marks

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X3 Prove that $\sec^2\theta + \operatorname{cosec}^2\theta \geq 4$ for all values of θ for which both sides are defined, and state the values of θ, $0 < \theta < \dfrac{\pi}{2}$, for which equality holds. [8 marks]

X4 In this question x is a real number with $0 < |x| \leq 1$.

(a) Show that $\tan(\arccos x) = \dfrac{\sqrt{1-x^2}}{x}$. [3 marks]

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(b) Hence solve $\tan(\arccos x) = 2x$. Give each answer in an exact form. [6 marks]

Answer $x = \ldots$

Total for X4: 9 marks

X5 A drone flies at a constant height of 40 m above flat ground. An observer stands on the ground. When the angle of elevation of the drone from the observer is θ radians, the distance from the observer to the drone is d m and the horizontal distance is h m, where

$$d = 40\,\mathrm{cosec}\,\theta, \quad h = 40\cot\theta, \quad 0 < \theta < \frac{\pi}{2}.$$

(a) Show that $d^2 - h^2 = 1600$. [2 marks]

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(b) Find d and θ when $h = 30$. [3 marks]

Answer $d = \ldots$, $\theta = \ldots$

(c) At one moment the distance d is 10 m more than h. Use part (a) to find h and θ at this moment. [4 marks]

Answer $h = \ldots$, $\theta = \ldots$

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(d) Give two reasons why the model may not give the true distance from the observer's eye to the drone. [2 marks]

Total for X5: 11 marks

X6 The curve C has equation $y = 2 + k \sec x$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$, $x \neq \frac{\pi}{2}$, where k is a positive constant.

(a) Show that C does not meet the x-axis when $k > 2$. [3 marks]

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(b) Given $k = 1$, C meets the x-axis at A and B. Find the exact distance AB. [3 marks]

Answer $AB = \ldots$

(c) Given $k = 1$, state the coordinates of the two turning points of C. [2 marks]

Answer .....

(d) Given $k = 1$, find the range of the function $y = 2 + \sec x$ on this domain. [2 marks]

Answer .....

Total for X6: 10 marks

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X7 In this question $x \neq n\pi$, where n is an integer.

(a) Prove that $(\operatorname{cosec} x - \cot x)^2 \equiv \dfrac{1 - \cos x}{1 + \cos x}$. [4 marks]

(b) Hence solve $(\operatorname{cosec} x - \cot x)^2 = \dfrac{1}{3}$ for $0 < x < 2\pi$, giving exact answers. [4 marks]

Answer $x = \ldots$

Total for X7: 8 marks

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X8 The functions f and g are defined by $f(x) = 2x - 1$, $x \in \mathbb{R}$, and $g(x) = \arctan x$, $x \in \mathbb{R}$.

(a) Solve $fg(x) = \frac{\pi}{2} - 1$. [2 marks]

Answer $x = \ldots$

(b) Find the range of fg. [2 marks]

Answer .....

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(c) Find $(fg)^{-1}(x)$ and state its domain. [4 marks]

Answer .....

Total for X8: 8 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can solve equations in sec, cosec and cot, including a changed interval.	○	○	○
I can prove an identity or inequality with reciprocal trig functions.	○	○	○
I can find inverse functions involving arcsin, arccos and arctan, with their domains.	○	○	○
I can use a trig model and say what is wrong with it.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Write $u = 2\theta - \dfrac{\pi}{3}$ and work out the interval u must lie in first.

Hint 2 Replace $\tan^2 u$ with $\sec^2 u - 1$ to get a quadratic in $\sec u$.

Hint 3 $\sec u = -\dfrac{3}{2}$ means $\cos u = -\dfrac{2}{3}$, which gives two values of u in the interval. Change each back to θ.

X2

Hint 1 On $0 < x \leq \dfrac{\pi}{2}$, $\sin x$ goes from 0 up to 1. What does $\operatorname{cosec} x$ do?

Hint 2 Write $y = 3 + 2\operatorname{cosec} x$ and make $\sin x$ the subject. The domain of f^{-1} is the range of f.

Hint 3 Use $\cot^2 x = \operatorname{cosec}^2 x - 1$. One root of the quadratic is not a possible value of $\operatorname{cosec} x$ here.

X3

Hint 1 Write $\sec^2\theta$ and $\operatorname{cosec}^2\theta$ with $\cos^2\theta$ and $\sin^2\theta$, then add the fractions.

Hint 2 The top becomes 1. So you need to show $\sin^2\theta\cos^2\theta \leq \dfrac{1}{4}$.

Hint 3 Put $s = \sin^2\theta$ and complete the square on $s(1-s)$. Do not divide by something that could be negative.

X4

Hint 1 Let $y = \arccos x$. Which interval is y in, and what sign does $\sin y$ have there?

Hint 2 Multiply both sides by x, then square. You get a quadratic in x^2.

Hint 3 Check a negative value of x in the original equation. Do not throw it away without checking.

X5

Hint 1 Square both formulas and use $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$.

Hint 2 $d^2 - h^2$ is a difference of two squares, and you know $d - h$.

Hint 3 Once you have h and d, $\sin\theta = \dfrac{40}{d}$. Your calculator must be in radians.

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X6

Hint 1 Set $y = 0$ and make $\sec x$ the subject. What values can $\sec x$ never take?

Hint 2 With $k = 1$, $\sec x = -2$ means $\cos x = -\dfrac{1}{2}$.

Hint 3 $\sec x$ has a minimum where $\cos x = 1$ and a maximum where $\cos x = -1$. Add 2.

X7

Hint 1 Write $\operatorname{cosec} x - \cot x$ as one fraction over $\sin x$.

Hint 2 Replace $\sin^2 x$ by $1 - \cos^2 x$ and factorise it.

Hint 3 In (b), set $\dfrac{1 - \cos x}{1 + \cos x} = \dfrac{1}{3}$ and solve for $\cos x$. There are two answers in the interval.

X8

Hint 1 $fg(x)$ means f applied to $g(x)$: it is $2 \arctan x - 1$.

Hint 2 The range of $\arctan$ is $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$. Double it and subtract 1.

Hint 3 The domain of $(fg)^{-1}$ is the range of fg.

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