A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 7: TRIGONOMETRY AND MODELLING · A* EXAM PRACTICE

Hints online, answers free with an account:
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Trigonometry and modelling: A* questions

85 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 Solve, for $0 \leq \theta < 360^\circ$, the equation [8 marks]

$$3\cos(\theta + 60^\circ) = 2\sin(\theta + 30^\circ).$$

Find the exact value of $\tan\theta$ first, then give each answer to 1 decimal place.

Answer $\tan\theta = \ldots$, $\theta = \ldots$

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X2

(a) Prove that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4 marks]

(b) By putting $x = \cos\theta$, solve $8x^3 - 6x - 1 = 0$. Give each root to 3 decimal places. [5 marks]

Answer $x = \ldots$

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(c) Hence show that $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \dfrac{1}{8}$. [3 marks]

Total for X2: 12 marks

X3 $f(x) = 5\sin x + 12\cos x$, where x is in radians.

(a) Write $f(x)$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [3 marks]
Give α to 3 decimal places.

Answer $f(x) = \ldots$

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(b) Find the set of values of k for which $f(x) = k$ has exactly two solutions in $0 \leq x \leq \pi$. [4 marks]

Answer $\ldots \leq k < \ldots$

(c) For all real values of x, find the greatest and least values of [4 marks]

$$g(x) = \frac{39}{20 - 5\sin x - 12\cos x},$$

and the smallest positive value of x at which the greatest value occurs.

Answer greatest $\ldots$, least $\ldots$, $x = \ldots$

Total for X3: 11 marks

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X4 Prove that [10 marks]

$$\tan\left(x+\frac{\pi}{4}\right)-\tan\left(x-\frac{\pi}{4}\right)\equiv 2\sec 2x.$$

Hence solve $\tan\left(x+\frac{\pi}{4}\right)-\tan\left(x-\frac{\pi}{4}\right)=4$ for $0<x<\pi$, giving exact answers.

Answer $x=\ldots$

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X5 A box has a rectangular cross-section 60 cm by 25 cm. It is tilted so that one corner touches a horizontal table and the 60 cm edge makes an angle θ with the table, where $0^\circ \leq \theta \leq 90^\circ$. The height, h cm, of the highest corner above the table is modelled by $h = 60\sin\theta + 25\cos\theta$.

(a) Explain why $h = 60\sin\theta + 25\cos\theta$. [2 marks]

(b) Write h in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$. Give α to 2 decimal places. [3 marks]

Answer $h = \ldots$

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(c) Find the greatest height of the highest corner and the value of θ at which it occurs. [2 marks]

Answer $h = \ldots$, $\theta = \ldots$

(d) The box is under a shelf 50 cm above the table. Find the set of values of θ for which the box fits under the shelf. [3 marks]

Answer $\cdots \leq \theta \leq \ldots$

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(e) Now θ is measured in radians and is small. Use small-angle approximations to show that $h \approx 25 + 60\theta - 12.5\theta^2$, and use this to estimate the value of θ for which $h = 30$. Compare it with the value from the model in part (b). [4 marks]

Answer $\theta \approx \ldots$

Total for X5: 14 marks

X6 $f(x) = 2\cos^2 x - \sin 2x$, where x is in radians.

(a) Show that $f(x) = 1 + \sqrt{2}\cos\left(2x + \frac{\pi}{4}\right)$. [4 marks]

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(b) Solve $f(x) = 0$ for $0 \leq x \leq \pi$. [3 marks]

Answer $x = \ldots$

(c) Find the set of values of k for which $f(x) = k$ has exactly two solutions in $0 \leq x \leq \pi$. [4 marks]

Answer $\cdots < k < \ldots$ or $\cdots < k < \ldots$

Total for X6: 11 marks

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X7 In triangle ABC, angle $B = 2\times$ angle A, $BC = 5$ cm and $AC = 8$ cm. [9 marks]
Find the exact length of AB. Do not use rounded values.

Answer $AB = \ldots$ cm

X8 In this question a, b and x are real numbers.

(a) Show that $\tan(\arctan a + \arctan b) = \dfrac{a+b}{1-ab}$, where $ab \neq 1$. [2 marks]

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(b) Show that $\arctan \frac{1}{2} + \arctan \frac{1}{3} = \frac{\pi}{4}$. [3 marks]

(c) Solve $\arctan x + \arctan 2x = \frac{\pi}{4}$. Give your answer in exact form and explain why any other root of your working is not a solution. [5 marks]

Answer $x = \ldots$

Total for X8: 10 marks

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F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use the addition and double-angle formulae to solve equations and prove identities.	○	○	○
I can write a sin x + b cos x as R sin(x + α) and use it for ranges and greatest values.	○	○	○
I can count the solutions of f(x) = k over an interval from the graph.	○	○	○
I can use a trigonometric model and test it with small-angle approximations.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1

Hint 1 Expand both sides with the addition formulae, using exact values for $30°$ and $60°$.

Hint 2 Collect the $\sin\theta$ terms on one side and the $\cos\theta$ terms on the other, then divide by $\cos\theta$.

Hint 3 $\tan$ repeats every $180°$, so there are two answers in the interval.

X2

Hint 1 Write $3x = 2x + x$ and use the addition formula for cosine, then the double-angle formulae.

Hint 2 $8x^3 - 6x = 2(4\cos^3\theta - 3\cos\theta)$. Use part (a). Three different values of θ in $0°$ to $180°$ give the three roots.

Hint 3 For $ax^3 + bx^2 + cx + d = 0$ the product of the roots is $-\dfrac{d}{a}$. Then use $\cos(180° - A) = -\cos A$.

X3

Hint 1 Expand $R\sin(x + \alpha)$ and compare the coefficients of $\sin x$ and $\cos x$.

Hint 2 Sketch $y = 13\sin(x + \alpha)$ for $0 \le x \le \pi$. Work out its value at each end of the interval and its highest point.

Hint 3 The fraction is greatest when the bottom is smallest. The bottom is $20 - f(x)$.

X4

Hint 1 Write $t = \tan x$ and use the addition formulae with $\tan\dfrac{\pi}{4} = 1$.

Hint 2 Put the two fractions over the common denominator $1 - t^2$ and simplify the top.

Hint 3 $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$. In the equation, solve for $2x$ in $0 < 2x < 2\pi$ first.

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X5

Hint 1 Draw the box with one corner on the table. Climb the long edge, then the short edge. The two edges meet at a right angle.

Hint 2 $R = \sqrt{60^2 + 25^2}$. For the fit under the shelf, think about which values of $\theta + \alpha$ are possible.

Hint 3 Use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$. The quadratic has two roots; only one is small.

X6

Hint 1 Replace $2\cos^2 x$ using a double-angle formula, then write $\cos 2x - \sin 2x$ as $R\cos(2x + \alpha)$.

Hint 2 Let $u = 2x + \dfrac{\pi}{4}$ and find the interval u lies in.

Hint 3 Sketch $\cos u$ over that interval. Look carefully at the value at the ends of the interval: it is reached three times.

X7

Hint 1 BC is opposite A and AC is opposite $B = 2A$. Use the sine rule and the double-angle formula to find $\cos A$.

Hint 2 The angles add to $180°$, so $C = 180° - 3A$ and $\sin C = \sin 3A$.

Hint 3 $\sin 3A = \sin(2A + A)$. Expand it, use your exact $\sin A$ and $\cos A$, then the sine rule once more.

X8

Hint 1 Let $P = \arctan a$, so $\tan P = a$. Then use the addition formula for $\tan(P + Q)$.

Hint 2 Knowing the tangent of an angle is 1 is not enough. Say which interval the sum must lie in.

Hint 3 Take tan of both sides and use part (a). Squaring is not needed, but taking tan can still bring in a root that does not work: check the sign.

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