A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 8: PARAMETRIC EQUATIONS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Parametric equations: A* questions

73 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The curve C has parametric equations [8 marks]

$$x = \cos 2t + 1, \quad y = 2\sin t, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}.$$

Find a Cartesian equation of C, stating the possible values of y. Hence find the set of values of the constant k for which the line $x + y = k$ meets C at two distinct points.

Answer $\cdots \leq k < \ldots$

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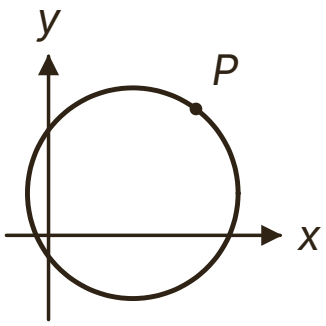
X2 The figure shows the circle C with parametric equations

$$x = 4 + 5\cos t, \quad y = 2 + 5\sin t, \quad 0 \leq t < 2\pi.$$

The point P on C has parameter $t = \alpha$, where $\tan\alpha = \frac{4}{3}$ and $0 < \alpha < \frac{\pi}{2}$.

(a) Find a Cartesian equation of C. [2 marks]



Answer .....

(b) Find the coordinates of P and show that the tangent to C at P has equation $3x + 4y = 45$. [4 marks]

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(c) The tangent at P meets the x-axis at Q. The centre of C is A . Find the area of triangle APQ. [3 marks]

Answer area $= \ldots$

Total for X2: 9 marks

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X3 The curve C has parametric equations [8 marks]

$$x = \sin t, \quad y = \sin 2t, \quad 0 \leq t < 2\pi.$$

Show that a Cartesian equation of C is $y^2 = 4x^2(1 - x^2)$. Find the coordinates of every point where C meets the line $y = x$, and state the values of t at which C passes through the origin.

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X4 A wheel of radius 0.4 m rolls along level ground. A mark M on the rim touches the ground at time $t = 0$. The position of M, in metres, t seconds later is modelled by

$$x = 0.4(5t - \sin 5t), \quad y = 0.4(1 - \cos 5t), \quad t \geq 0.$$

(a) Find the height of M above the ground when $t = 1$. [2 marks]

Answer ... m

(b) Find the time when M is first at its highest point, and its horizontal distance from the start at that time. [3 marks]

Answer .....

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(c) Find, in seconds, how long M spends more than 0.6 m above the ground in each turn of the wheel. [3 marks]

Answer ... s

(d) State two assumptions made in this model. [2 marks]

Total for X4: 10 marks

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X5 The curve C has parametric equations

$$x = \frac{1 - t^2}{1 + t^2}, \quad y = \frac{2t}{1 + t^2}, \quad t \in \mathbb{R}.$$

(a) Show that every point of C lies on the circle $x^2 + y^2 = 1$. [3 marks]

(b) Prove that $t = \dfrac{y}{1 + x}$ for every point of C, and explain why no value of t gives the point $(-1, 0)$. [3 marks]

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(c) Find the value of t at the point $\left(\frac{3}{5}, \frac{4}{5}\right)$, and the coordinates of the point where $t = 3$. [3 marks]

Answer .....

Total for X5: 9 marks

X6 The ellipse E has parametric equations

$$x = 3\cos t, \quad y = 2\sin t, \quad 0 \leq t < 2\pi.$$

(a) Find a Cartesian equation of E. [2 marks]

Answer .....

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(b) The line $y = mx + 4$ touches E. Show that $3m^2 = 4$. [4 marks]

(c) Find the exact coordinates of the points where the two tangents touch E, and the value of t at each. [4 marks]

Answer .....

Total for X6: 10 marks

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X7 The curve C has parametric equations [8 marks]

$$x = 3\sec t, \quad y = 2\tan t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

Find a Cartesian equation of C and state the possible values of x. Show that the line $2x - 3y = 6$ meets C at exactly one point, and find its coordinates and the value of t there. Show also that C never meets the line $2x = 3y$.

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X8 A stone is thrown from the top of a wall 20 m high, above level ground. Its position, in metres, t seconds after it is thrown is modelled by

$$x = 24t, \quad y = 20 + 18t - 5t^2,$$

where x is measured horizontally from the foot of the wall and y is the height above the ground.

(a) Find the greatest height of the stone and its horizontal distance from the wall at that moment. [3 marks]

Answer .....

(b) Find how far from the wall the stone lands. [3 marks]

Answer ... m

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(c) Find the length of time for which the stone is more than 30 m above the ground. [3 marks]

Answer ... s

(d) Give two criticisms of the model. [2 marks]

Total for X8: 11 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find a Cartesian equation and state the values the variables can take.	○	○	○
I can find where a parametric curve meets a line, including tangency conditions.	○	○	○
I can prove a fact about a parametric curve and use it.	○	○	○
I can use a parametric model and criticise it.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Write $\cos 2t$ in terms of $\sin t$, then replace $\sin t$ with $\frac{y}{2}$. The range of t gives the range of y.
	Hint 2	Put $x = k - y$ into your Cartesian equation and get a quadratic in y.
	Hint 3	Two distinct roots is not enough: both roots must be values of y that are on C. Check the larger root against $y \leq 2$.
X2	**Hint 1**	Make $\cos t$ and $\sin t$ the subjects, then square and add.
	Hint 2	Draw a right-angled triangle with sides 3 and 4 to find $\cos\alpha$ and $\sin\alpha$. A tangent is perpendicular to the radius.
	Hint 3	The angle between a radius and the tangent is $90°$, so AP and PQ are the two shorter sides of the triangle.
X3	**Hint 1**	Write $\sin 2t$ as $2\sin t\cos t$ and square it.
	Hint 2	Solve $y = x$ in terms of t, not in x and y: $\sin 2t = \sin t$. Do not divide by $\sin t$.
	Hint 3	Work out x and y from each value of t. Two different values of t give the origin.
X4	**Hint 1**	Your calculator must be in radians. $5t$ is an angle.
	Hint 2	y is greatest when $\cos 5t$ is least. In (c), solve $\cos 5t = -\frac{1}{2}$ for $0 \leq 5t \leq 2\pi$.
	Hint 3	In (c), the angle $5t$ is between the two values you found. Divide the gap by 5 to change it into seconds.
X5	**Hint 1**	Put $x^2 + y^2$ over the common denominator $(1+t^2)^2$ and expand the top.
	Hint 2	Work out $1 + x$ as one fraction first. For $(-1, 0)$, set $x = -1$ and see what the equation says.
	Hint 3	Part (b) gives t straight from x and y.

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X6

Hint 1 Make $\cos t$ and $\sin t$ the subjects and use $\cos^2 t + \sin^2 t = 1$.

Hint 2 A line touches a curve when the quadratic you get has one repeated root.

Hint 3 A repeated root is $x = -\dfrac{b}{2a}$. Then use $y = 2\sin t$ and the sign of x to choose t.

X7

Hint 1 Use $\sec^2 t = 1 + \tan^2 t$. On this interval $\cos t > 0$.

Hint 2 Substitute $y = \dfrac{2x - 6}{3}$. The x^2 terms cancel, so there is only one solution.

Hint 3 For $2x = 3y$, substitute $y = \dfrac{2x}{3}$ and see what is left.

X8

Hint 1 Only y matters for the height. Complete the square on $20 + 18t - 5t^2$.

Hint 2 The stone lands when $y = 0$. Only the positive root of the quadratic makes sense. Then find x.

Hint 3 In (c), solve $y = 30$. The stone is above 30 m between the two times.

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