A-LEVEL MATHS (EDEXCEL 9MA0) · PURE YEAR 2 · CHAPTER 9: DIFFERENTIATION · A* EXAM PRACTICE

Hints online, answers free with an account:
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Differentiation: A* questions

81 marks

Date

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X A* exam questions

A to A*

Extended Edexcel-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 The curve C has equation [9 marks]

$$x^2 - 2xy + 3y^2 = 18.$$

Find the exact coordinates of the points on C where the tangent is parallel to the y-axis.

Answer points ...

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X2 The curve C has parametric equations

$$x = \tan t, \quad y = \sin 2t, \quad 0 \leq t < \frac{\pi}{2}.$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\cos^2 t \cos 2t$. [3 marks]

(b) Find the exact coordinates of the stationary point of C. [3 marks]

Answer $(\ldots, \ldots)$

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(c) The tangent to C at the point where $t = \frac{\pi}{6}$ meets the x-axis at A. Find the exact x-coordinate of A. [4 marks]

Answer $x = \ldots$

(d) Show that a Cartesian equation of C is $y = \frac{2x}{1 + x^2}$ and state the set of values of x for this curve. [3 marks]

Total for X2: 13 marks

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X3 The curve C has equation $y = (x^2 + k)e^x$, where k is a constant.

(a) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = e^x(x^2 + 4x + k + 2)$. [3 marks]

(b) Find the set of values of k for which C has two points of inflection. Explain why C has no point of inflection when $k = 2$. [3 marks]

(c) In the rest of this question $k = -8$. Find the exact set of values of x for which C is concave. [3 marks]

Answer .....

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(d) Find the x-coordinates of the stationary points of C and determine their nature. [3 marks]

Answer .....

Total for X3: 12 marks

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X4 A bowl is a hemisphere of radius 10 cm. Water is poured in at a constant rate of $40\ \text{cm}^3$ per second. When the depth of the water is h cm, the volume of water is [8 marks]

$$V = \frac{\pi h^2(30 - h)}{3}.$$

Find the rate at which the depth is increasing when $h = 4$, and the rate at which the area of the water's surface is increasing at that moment. Give each answer to 3 significant figures.

Answer depth: ... ; area: ...

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X5 In this question x is in radians.

(a) Prove from first principles that the derivative of $\sin x$ is $\cos x$. You may assume that, as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$. [5 marks]

(b) The curve C has equation $y = 3^x \sin x$. Find the smallest positive value of x at which C has a stationary point. Give your answer to 3 significant figures. [5 marks]

Answer $x = \ldots$

Total for X5: 10 marks

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X6 The curve C has equation $x = ye^{2y}$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. [3 marks]

Answer .....

(b) Find the exact coordinates of the point on C where the tangent is parallel to the y-axis. [2 marks]

Answer $(\ldots, \ldots)$

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(c) The normal to C at the point where $y = 1$ meets the x-axis at P. Find the exact x-coordinate of P. [4 marks]

Answer $x = \ldots$

Total for X6: 9 marks

X7 The number of bees in a colony is modelled by

$$P = \frac{2000}{1 + 9e^{-0.4t}},$$

where t is the time in weeks after the colony was set up.

(a) Show that $\dfrac{\mathrm{d}P}{\mathrm{d}t} = 0.4P\left(1 - \dfrac{P}{2000}\right)$. [4 marks]

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(b) Find the exact time at which the number of bees is increasing fastest, and the rate of increase at that time. [4 marks]

Answer $t = \ldots$ weeks; rate $\ldots$

(c) Give one way in which the model may not fit a real colony, and say what happens to P in the model as t becomes large. [2 marks]

Total for X7: 10 marks

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X8 The function f is defined by $f(x) = \dfrac{\ln x}{x^k}$, $x > 0$, where k is a positive constant.

(a) Show that $f'(x) = \dfrac{1 - k\ln x}{x^{k+1}}$. [3 marks]

(b) Show that the greatest value of $f(x)$ is $\dfrac{1}{ke}$. [3 marks]

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(c) Hence, by taking $k = 1$, prove that $e^{\pi} > \pi^{e}$. [4 marks]

Total for X8: 10 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find where a tangent is vertical on a parametric, implicit or x = f(y) curve.	○	○	○
I can decide the points of inflection of a curve with an unknown constant.	○	○	○
I can link rates of change with the chain rule, including a length found by Pythagoras.	○	○	○
I can use a stationary value to prove an inequality.	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Differentiate every term with respect to x. The $2xy$ term needs the product rule.
	Hint 2	A tangent parallel to the y-axis has an infinite gradient. Which part of your fraction for the gradient must be 0?
	Hint 3	Put $x = 3y$ into the equation of C. There are two points.
X2	**Hint 1**	Find $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t}$, then divide. Dividing by $\sec^2 t$ is multiplying by $\cos^2 t$.
	Hint 2	For the tangent you need a point and a gradient, both from $t = \dfrac{\pi}{6}$.
	Hint 3	$\sin 2t = 2\sin t \cos t$. Divide top and bottom by $\cos^2 t$ to bring in $\tan t$.
X3	**Hint 1**	Use the product rule twice. Take e^x out as a common factor each time.
	Hint 2	e^x is always positive, so only the quadratic decides the sign. A point of inflection needs the sign to change.
	Hint 3	Concave means the second derivative is negative. Use it again for the nature of the stationary points.
X4	**Hint 1**	Differentiate V with respect to h, then use $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}t} \div \dfrac{\mathrm{d}V}{\mathrm{d}h}$.
	Hint 2	Draw the bowl from the side. The centre of the hemisphere is $10 - h$ above the water surface, and the rim of the surface is 10 from the centre.
	Hint 3	Write the area A in terms of h, then $\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}h} \times \dfrac{\mathrm{d}h}{\mathrm{d}t}$.
X5	**Hint 1**	Start from the gradient of a chord, $\dfrac{f(x+h) - f(x)}{h}$, and use the addition formula for $\sin(x + h)$.
	Hint 2	Write 3^x as $e^{x \ln 3}$ to differentiate it, then use the product rule.
	Hint 3	$\tan x$ is negative. The calculator gives a negative angle: the first positive solution is π less than that number's size.
X6	**Hint 1**	Here x is given in terms of y. Differentiate x with respect to y, then turn the fraction upside down.
	Hint 2	A tangent parallel to the y-axis is where $\dfrac{\mathrm{d}x}{\mathrm{d}y} = 0$.
	Hint 3	The normal's gradient is -1 divided by the tangent's gradient.

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X7

Hint 1 Write $P = 2000(1 + 9e^{-0.4t})^{-1}$ and use the chain rule. Then write the right-hand side of the result in terms of t too.

Hint 2 Part (a) gives the rate as a quadratic in P. Find the value of P that makes it greatest.

Hint 3 As t grows, $e^{-0.4t}$ goes to 0.

X8

Hint 1 Use the quotient rule. The derivative of x^k is kx^{k-1}.

Hint 2 Solve $f'(x) = 0$ and check that $f'(x)$ changes from positive to negative there.

Hint 3 Put $x = \pi$ into the inequality $\dfrac{\ln x}{x} \leq \dfrac{1}{e}$, then take $\ln$ of e^{π} and of π^{e}.

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