

Quadratic graphs and the discriminant

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-quadratic-graphs-and-the-discriminant/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Write $x^2 - 8x + 23$ in the form $(x + p)^2 + q$.
- A2** Needed today [2 marks] Solve $3x^2 - 5x - 2 = 0$.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer

- A3** Last lesson [2 marks] Find the four roots of $f(x) = x^4 - 10x^2 + 9$.
- A4** From chapter 1 [2 marks] Rationalise the denominator of $\frac{6}{3 - \sqrt{3}}$.

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Sketch $y = x^2 + 2x - 35$. Show the axis crossings and the turning point.

Hint Factorise. The turning point is half-way between the roots.

B1 Sketch $y = 2x^2 - x - 6$, [3 marks]
showing where it meets the axes and its turning point.

EXAMPLE 2

Sketch the graph of $y = 4x - x^2 - 7$. Give the coordinates of the turning point and show where the graph meets the axes.

Hint Write it as $-x^2 + 4x - 7$ and take out a factor of -1 before completing the square. Check the discriminant to see if it meets the x -axis.

B2 Write $6x - 3x^2 - 5$ in [4 marks]
the form $a(x + p)^2 + q$.
Hence sketch
 $y = 6x - 3x^2 - 5$,
showing its turning point and where it meets the axes.

EXAMPLE 3

The graph of $y = ax^2 + bx + c$ meets the x -axis at $(-1, 0)$ and $(4, 0)$ and the y -axis at $(0, -8)$. Find a , b and c .

Hint If the roots are -1 and 4 , then $y = a(x + 1)(x - 4)$. Use the y -intercept to find a .

B3 The graph of $y = ax^2 + bx + c$ has a maximum point at $(-2, 7)$ and passes through $(0, -5)$. Find a , b and c . [3 marks]

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

EXAMPLE 4

Find the values of k for which $(k - 1)x^2 + 6x + (k + 7) = 0$ has equal roots.

Hint Equal roots means $b^2 - 4ac = 0$. Here $a = k - 1$, $b = 6$ and $c = k + 7$. You get a quadratic in k .

B4 Find the values of k for which $x^2 + (k + 3)x + 4k = 0$ has equal roots. [3 marks]

Answer $k = \dots\dots$ or $k = \dots\dots$

EXAMPLE 5

Find the range of values of k for which $3x^2 - 6x + k = 0$ has two distinct real roots.

Hint Two distinct real roots means $b^2 - 4ac > 0$. The answer is an inequality.

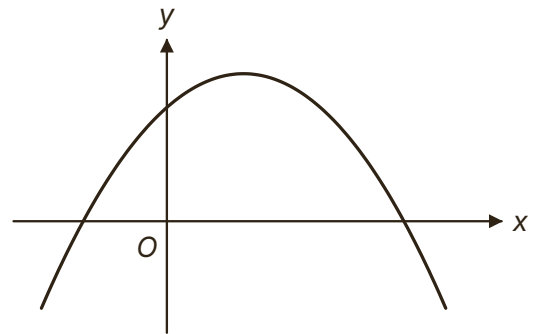
B5 Find the range of values of k for which $x^2 - 4x + (2k - 1) = 0$ has no real roots. [3 marks]

Answer $k \dots\dots$

- C1** $f(x) = 7 + 3x - 2x^2$. Find the value of the discriminant of $f(x)$, and [2 marks]
hence state how many times the graph of $y = f(x)$ meets the x -axis.

Answer

- C2** The sketch shows the graph of $y = ax^2 + bx + c$. State, with a reason [4 marks]
in each case, whether each of a , b , c and $b^2 - 4ac$ is positive,
negative or zero.



- C3** Find the range of values of k for which the line $y = k$ does not meet [3 marks]
the curve $y = x^2 - 10x + 33$.

Answer $k \dots\dots$

D1 Leah writes: $3x^2 - 18x + 22 = 3(x - 3)^2 + 13$, so the minimum point of $y = 3x^2 - 18x + 22$ is $(3, 13)$.

$f(x) = 3x^2 - 18x + 22$. Leah is wrong.

(a) Describe the mistake Leah has made. [2 marks]

(b) Write $f(x)$ in the form $a(x + p)^2 + q$, where a , p and q are integers. Hence write down the coordinates of the turning point of $y = f(x)$ and the equation of its line of symmetry. [5 marks]

Answer $f(x) = \dots\dots\dots$ $(\dots\dots\dots, \dots\dots\dots)$ $x = \dots\dots\dots$

(c) Hence solve $f(x) = 0$, giving your answers in the form $\frac{a \pm \sqrt{b}}{c}$, [2 marks]
where a , b and c are integers.

Answer $x = \dots\dots\dots$

Total for D1: 9 marks

D3 $h(x) = 3x^2 + (k - 1)x - (k + 2)$, where k is a real constant.

- (a) Find the discriminant of $h(x)$ in terms of k , simplifying your answer. [3 marks]

Answer

- (b) Hence prove that $h(x)$ has real roots for every value of k . [2 marks]

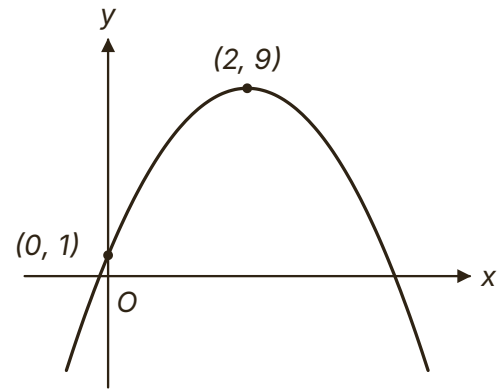
- (c) Find the value of k for which $h(x)$ has equal roots, and find this repeated root. [3 marks]

Answer $k = \dots\dots$ $x = \dots\dots$

Total for D3: 8 marks

D4 The sketch shows $y = ax^2 + bx + c$, with maximum point $(2, 9)$ and y -intercept $(0, 1)$.

(a) Find the values of a , b and c . [4 marks]



Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

(b) Hence find the exact x -coordinates of the points where the graph meets the x -axis. Give your answers in the form $p \pm q\sqrt{2}$, where p and q are rational. [3 marks]

Answer $x = \dots\dots$

Total for D4: 7 marks

D5 Given that $y = 2^x$,

(a) show that the equation $4^x - k \times 2^{x+1} + 16 = 0$ can be written as $y^2 - 2ky + 16 = 0$, and hence find the values of k for which this equation in y has equal roots. [4 marks]

Answer $k = \dots\dots$

- (b) For one value of k the equation in x has one solution. Find it, and [3 marks]
explain why the other value of k gives none.

Answer $x = \dots\dots$

Total for D5: 7 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Find every integer k for which the equation $x^2 + kx + k + 8 = 0$ [6 marks]
has two distinct roots that are both integers.

Answer $k = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Call the roots r and s , so
 $x^2 + kx + k + 8 = (x - r)(x - s)$.
- Hint 2** $r + s = -k$ and $rs = k + 8$. Add these:
 $rs + r + s = 8$, so $(r + 1)(s + 1) = 9$.
- Hint 3** Use the pairs 1×9 and $(-1) \times (-9)$. The
pairs 3×3 and $(-3) \times (-3)$ give equal roots.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch a quadratic and its turning point. ☐ ☐ ☐

I can find a quadratic from its sketch. ☐ ☐ ☐

I can count real roots with the discriminant. ☐ ☐ ☐

I can find k using the discriminant. ☐ ☐ ☐