Radian measure and arc length

112 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-radian-measure-and-arc-length/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Write 135° in radians as a multiple of π.

Answer .....

A2 Common slip [2 marks]

A student works out $\cos 2$ on a calculator in degree mode and gets 0.9994. Find $\cos 2$, where the angle is 2 radians, to 3 decimal places.

Answer .....

A3 Needed today [2 marks]

An arc of a circle of radius 9 cm subtends an angle of 40° at the centre. Find the exact length of the arc, using a fraction of the circumference.

Answer .....

A4 Needed today [2 marks]

Find the exact value of $\tan 60^\circ \times \cos 30^\circ$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

(a) Write 210° in radians, as a multiple of π. (b) Write 1.2 radians in degrees, to 1 decimal place.

Hint $180^\circ = \pi$ radians. Multiply by $\frac{\pi}{180}$ to go to radians, and by $\frac{180}{\pi}$ to go to degrees.

B1 (a) Write 330° in radians, as a multiple of π. (b) Write 2.5 radians in degrees, to 1 decimal place. [3 marks]

Answer .....

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EXAMPLE 2

A sector of a circle has radius 6 cm and arc length 7.5 cm. Find the angle of the sector, in radians, and the perimeter of the sector.

Hint Use $s = r\theta$ with θ in radians. The perimeter is the arc and two radii.

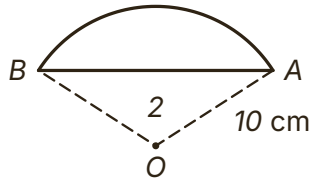
B2 A sector has angle 0.8 radians and perimeter 21 cm. Find the radius and the arc length. [4 marks]

Answer .....

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EXAMPLE 3

The chord AB of a circle with centre O and radius 10 cm subtends an angle of 2 radians at O. Find the perimeter of the minor segment cut off by AB, to 3 significant figures.



Hint The segment is bounded by the arc and the chord. Split triangle OAB into two right-angled triangles to find the chord.

B3 The chord PQ of a circle with centre O and radius 8 cm subtends an angle of 1.4 radians at O. Find the perimeter of the minor segment cut off by PQ, to 3 significant figures. [5 marks]

Answer .....

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EXAMPLE 4

A circle with centre O has radius 5 cm. The chord AB has length 8 cm. Find the angle AOB in radians, to 3 decimal places, and the length of the minor arc AB.

Hint Use the cosine rule in triangle OAB, with the calculator in radian mode.

B4 A circle with centre O has radius 13 cm. The chord AB has length 10 cm. Find (a) the angle AOB in radians, to 3 decimal places, (b) the length of the minor arc AB, (c) the perimeter of the minor segment. [6 marks]

Answer .....

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EXAMPLE 5

The minute hand of a clock is 12 cm long. Find the exact distance moved by its tip in 40 minutes.

Hint One hour is a full turn, 2π radians. Find the angle first.

B5 The minute hand of a clock is 15 cm long. Find the exact distance moved by its tip in 25 minutes. [3 marks]

Answer .....

C Practice

A-Level

C1 Ola finds the arc length of a sector of radius 6 cm and angle $40°$. She writes: "arc $= r\theta = 6 \times 40 = 240$ cm". Find her mistake and give the correct arc length exactly. [4 marks]

Answer .....

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C2 The string of a pendulum is 80 cm long. The pendulum swings 0.3 radians either side of the vertical. Find (a) the length of the arc the bob travels in one swing from one end to the other, (b) the height the bob rises above its lowest point, to 3 significant figures. [5 marks]

Answer .....

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C3 A shape is bounded by two arcs with the same centre, of radii 8 cm and 5 cm, both with angle θ radians, and two straight edges of length 3 cm. The perimeter of the shape is 30 cm. Find the exact value of θ. [5 marks]

Answer .....

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C4 Ben says that the perimeter of a sector of radius 9 cm and angle $\frac{2\pi}{3}$ is 6π cm. Find his mistake and give the correct perimeter, exactly and to 3 significant figures. [4 marks]

Answer .....

C5 Two sectors each have perimeter 30 cm. Sector P has radius 6 cm. Sector Q has angle 1.5 radians. Find the angle of P, the radius of Q, and the exact difference between their arc lengths. [5 marks]

Answer .....

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D1 A sector OAB of a circle has radius r cm and angle θ radians. Its perimeter is 40 cm.

(a) Show that $\theta = \dfrac{40 - 2r}{r}$. [2 marks]

(b) The arc AB has length 16 cm. Find r and θ. [3 marks]

Answer .....

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(c) Find the set of possible values of r, giving the ends of the interval exactly. [4 marks]

Answer .....

Total for D1: 9 marks

D2

A circle with centre O has radius 6 cm. The chord AB has length $6\sqrt{3}$ cm. Zara says: "Angle AOB is $120°$, so the minor arc AB is $r\theta = 6 \times 120 = 720$ cm."

Zara is wrong.

(a) Explain her mistake. [2 marks]

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(b) Show that angle $AOB = \dfrac{2\pi}{3}$ and find the exact length of the minor arc AB. [3 marks]

Answer .....

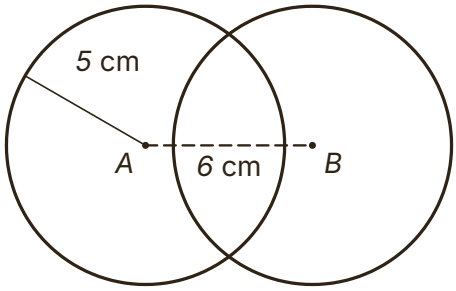
(c) Find the exact perimeter of the major segment cut off by AB. [3 marks]

Answer .....

Total for D2: 8 marks

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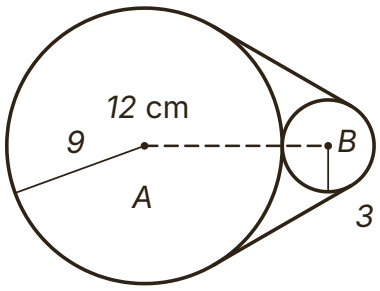
D3 Two circles each have radius 5 cm. Their centres A and B are 6 cm apart. Find the perimeter of the region that is inside both circles, to 3 significant figures. [7 marks]



Answer .....

D4 A belt passes tightly round two pulleys of radii 9 cm and 3 cm. The centres A and B of the pulleys are 12 cm apart. The straight parts of the belt touch both pulleys.

(a) Show that each straight part of the belt has length $6\sqrt{3}$ cm. [2 marks]



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(b) Show that the belt is in contact with the larger pulley for an angle of $\frac{4\pi}{3}$ radians. [3 marks]

(c) Find the exact length of the belt. [5 marks]

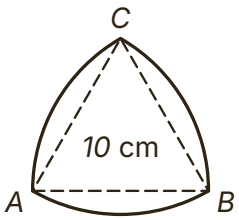
Answer .....

Total for D4: 10 marks

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D5 Shape 1 is made from an equilateral triangle ABC of side 10 cm. Each pair of vertices is joined by an arc whose centre is the third vertex, as in the diagram. Shape 2 is made in the same way from a regular pentagon of side 6 cm: each pair of next-door vertices is joined by an arc whose centre is the vertex opposite them.

(a) Find the exact perimeter of Shape 1. [3 marks]



Answer .....

(b) In the pentagon of Shape 2, each vertex is joined to the two vertices opposite it. Show that the angle between these two lines is $\frac{\pi}{5}$. [3 marks]

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(c) Find the perimeter of Shape 2, to 3 significant figures. [3 marks]

Answer .....

Total for D5: 9 marks

D6 A wheel of radius 30 cm rolls along level ground without slipping. A point P on the rim is touching the ground at the start.

(a) The wheel rolls 1 m. Find the angle, in radians, through which it turns. [2 marks]

Answer .....

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(b) Find the height of P above the ground at this moment, to 3 significant figures. [3 marks]

Answer .....

(c) Find the exact distance the wheel rolls before P is first 45 cm above the ground. [3 marks]

Answer .....

Total for D6: 8 marks

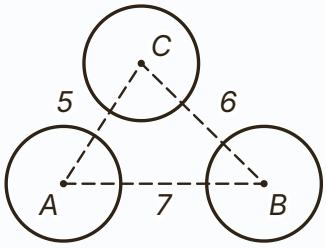
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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Three circles each have radius 2 cm. Their centres A, B and C are the corners of a triangle with sides 5 cm, 6 cm and 7 cm. A band is stretched tightly round the outside of all three circles. Show that the length of the band is $18 + 4\pi$ cm. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Each straight part of the band touches two circles of the same radius. How long is it, compared with a side of the triangle?

Hint 2 At each circle the band follows an arc. The radii at the two ends of that arc are perpendicular to the two sides that meet at that centre.

Hint 3 The arc at a corner turns through the exterior angle of the triangle there. What do the three exterior angles of a triangle add up to?

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can change an angle between degrees and radians, exactly or to a given accuracy.	○ ○ ○
I can use $s = r\theta$ with θ in radians to find an arc length, a radius or an angle.	○ ○ ○
I can find the perimeter of a sector or a segment, including the straight edges.	○ ○ ○
I can find an angle at the centre from a chord, using the cosine rule or right-angled triangles.	○ ○ ○

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