Rates of change

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-rates-of-change/

Date

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A Do Now

Quick questions to start.

A1 Needed today [2 marks]

The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

Find $\frac{dV}{dr}$.

Answer .....

A2 Needed today [2 marks]

Differentiate $(2x + 1)^4$ with respect to x.

Answer .....

A3 Needed today [2 marks]

Given $y = \frac{3}{x^2}$, find $\frac{dy}{dx}$.

Answer .....

A4 Common slip [2 marks]

Water drains out of a tank at 5 cm³ per second. The volume of water in the tank is V cm³ at time t seconds.

Write down $\frac{dV}{dt}$, and say why its sign matters.

Answer .....

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A5 From chapter 9 [2 marks]

A cone has height h and base radius r, where $r = \frac{h}{3}$. Write the volume $V = \frac{1}{3}\pi r^2 h$ in terms of h only.

Answer .....

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The radius of a circle increases at 2 cm s^{-1}. Find the rate at which the area is increasing when the radius is 5 cm.

Hint Write $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$. You know $\frac{dr}{dt}$; find $\frac{dA}{dr}$ from $A = \pi r^2$.

B1 The edge of a cube increases at 0.3 cm s^{-1}. Find the rate at which the surface area of the cube is increasing when the edge is 4 cm. [4 marks]

Answer .....

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EXAMPLE 2

Air is pumped into a spherical balloon at 10 cm³ s⁻¹. Find the rate at which the radius is increasing when the radius is 5 cm.

Hint This time you know $\frac{dV}{dt}$ and want $\frac{dr}{dt}$. Divide by $\frac{dV}{dr}$.

B2 Air is pumped into a spherical balloon at 20 cm³ s⁻¹. When the radius is 5 cm, find the rate at which the radius is increasing, and the rate at which the surface area $S = 4\pi r^2$ is increasing. [5 marks]

Answer .....

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EXAMPLE 3

An inverted cone has height 8 cm and top radius 4 cm. Water is poured in at 3 cm³ s⁻¹. Find the rate at which the depth is rising when the depth is 4 cm.

Hint By similar triangles the water surface has radius $r = \frac{h}{2}$. Write V in terms of h only, then differentiate.

B3 An inverted cone has height 10 cm and top radius 15 cm. Water leaks out of it at 2 cm³ s⁻¹. Find the rate of change of the depth h cm when $h = 4$. [5 marks]

Answer .....

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EXAMPLE 4

Water leaks from a tank. The depth h m falls at a rate proportional to the square root of the depth. Write a differential equation for h.

Hint "Proportional to" means a constant k times the quantity. Choose the sign so that $k > 0$.

B4 A circular patch of mould grows so that the rate of increase of its radius r is inversely proportional to r. Show that the area of the patch increases at a constant rate. [4 marks]

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EXAMPLE 5

Liquid is poured into a tank at 20 cm³ s⁻¹. It leaks out at $0.1V$ cm³ s⁻¹, where V cm³ is the volume in the tank. Form a differential equation for V.

Hint Rate of change = rate in minus rate out.

B5 Water is poured into a tank at 12 cm³ s⁻¹, and leaks out at $\frac{V}{50}$ cm³ s⁻¹, where V cm³ is the volume in the tank. Form a differential equation for V, and find the volume at which it stops changing. [4 marks]

Answer .....

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C1 The area of a square increases at a constant 6 cm² s^{-1}. When the area is 36 cm², find the rate at which the side length is increasing, and the rate at which the perimeter is increasing. [5 marks]

Answer .....

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C2 Water is poured at 30 cm³ s⁻¹ into a cylinder of radius 5 cm standing on its base. Find the exact rate at which the depth of the water rises. [4 marks]

Answer .....

C3 The area of a circular oil slick is decreasing at 3 cm² s⁻¹. Lee works out the rate of change of the radius when the radius is 2 cm and gets $\frac{3}{4\pi}$. Explain what is wrong with Lee's answer, and give the correct value. [4 marks]

Answer .....

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C4 The rate of increase of a quantity x is inversely proportional to x^2. When $x = 2$, x is increasing at 3 units per second. Find the rate of increase of x when $x = 3$. [4 marks]

Answer .....

C5 The surface area of a cube increases at 6 cm² s⁻¹. Find the rate at which its volume increases when the edge is 3 cm. [5 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1 Oil spreads on calm water as a thin circular slick of radius r m and constant thickness h m. The volume of oil is fixed at 200 m³, and the radius increases at 0.5 m s⁻¹.

(a) Find the rate at which the area of the slick increases when $r = 20$. [2 marks]

Answer .....

(b) Show that $h = \dfrac{200}{\pi r^2}$, and find the exact rate of change of the thickness when $r = 20$. [4 marks]

Answer .....

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(c) Find the thickness of the slick at the moment its area is increasing at 30π m² s⁻¹. [3 marks]

Answer .....

Total for D1: 9 marks

D2

Water drains from an inverted cone of height 30 cm and top radius 10 cm at a constant 5 cm³ s⁻¹. Jo says: "The water leaves at a constant rate, so the depth falls at a constant rate too."

Jo is wrong.

(a) Show that when the depth is h cm, the volume of water is $V = \dfrac{\pi h^3}{27}$. [2 marks]

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(b) Show that $\frac{dh}{dt} = -\frac{45}{\pi h^2}$. [3 marks]

(c) Find the exact rate at which the depth is falling when $h = 15$ and when $h = 7.5$. [3 marks]

Answer .....

(d) Explain why Jo is wrong. [2 marks]

Total for D2: 10 marks

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D3 A spherical balloon is inflated so that its surface area increases at a constant 40 cm² s⁻¹. Find the rate at which its volume is increasing when the radius is 5 cm. [6 marks]

Answer .....

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D4 Water flows at 15 cm³ s⁻¹ into a tall cylindrical tank whose base has area 50 cm². Water leaks out through a hole in the base at a rate of kh cm³ s⁻¹, where h cm is the depth of the water and k is a constant.

(a) Show that $\dfrac{dh}{dt} = \dfrac{15 - kh}{50}$. [3 marks]

(b) When $h = 2$, the depth is rising at 0.18 cm s⁻¹. Find k. [2 marks]

Answer .....

(c) Find the depth at which the water level stays constant. [2 marks]

Answer .....

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(d) The tank starts empty. Show that $h = 5 - 5e^{-\frac{3t}{50}}$ satisfies the differential equation, and find the rate at which the depth rises at the start. [3 marks]

Answer .....

Total for D4: 10 marks

D5 A point P moves along the curve $y = x^3 - 2x$. Its x-coordinate increases at a constant 3 units per second.

(a) Find the rate of change of the y-coordinate of P when $x = 2$. [2 marks]

Answer .....

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(b) Find the exact values of x at which the y-coordinate of P is decreasing at 3 units per second. [3 marks]

Answer .....

(c) Find the rate of change of the gradient of the curve at P when $x = 2$. [3 marks]

Answer .....

Total for D5: 8 marks

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D6 A cube of ice melts. Its volume decreases at a rate proportional to its surface area. The edge is x cm after t minutes.

(a) Show that $\dfrac{dx}{dt}$ is constant. [3 marks]

(b) The edge is 3 cm at the start and 2 cm after 10 minutes. Find how long the cube takes to melt completely. [2 marks]

Answer .....

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(c) Find the rate of change of the volume when the edge is 2.5 cm. [3 marks]

Answer .....

Total for D6: 8 marks

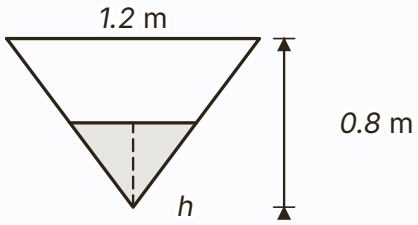
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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A water trough is 2 m long. Its cross-section is an isosceles triangle, 1.2 m across the top and 0.8 m deep, with the vertex at the bottom. Water flows in at 0.003 m³ s⁻¹ and leaks out at $0.009h$ m³ s⁻¹, where h m is the depth. Find the rate at which the area of the water surface is increasing when $h = 0.2$, and the depth that the water approaches. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 By similar triangles, the water surface is $1.5h$ m wide. Write the volume in terms of h.

Hint 2 $V = 1.5h^2$, so $\frac{dV}{dh} = 3h$, and $\frac{dV}{dt} = 0.003 - 0.009h$. Divide to get $\frac{dh}{dt}$.

Hint 3 The water surface is a rectangle 2 m by $1.5h$ m. Differentiate its area with respect to t. The depth approached is where $\frac{dh}{dt} = 0$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the chain rule to link two rates of change, such as dV/dt and dr/dt. ○ ○ ○

I can write a volume or area in terms of one variable before differentiating. ○ ○ ○

I can give a rate the right sign when a quantity is decreasing. ○ ○ ○

I can form a differential equation from a statement about a rate of change. ○ ○ ○

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