

Reciprocal graphs and points of intersection

70 marks



Hints online, answers free
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mathswariz.uk/worksheets/a-level-reciprocal-graphs-and-points-of-intersection/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Solve $4x^2 + 5x - 6 = 0$.

Answer
- A2** Last lesson [2 marks] Write down the coordinates of the points where $y = (x - 3)(x + 2)^2$ meets the axes.

Answer

- A3** From chapter 2 [2 marks] Find the set of values of k for which $x^2 + 8x + k = 0$ has no real roots.

Answer
- A4** Needed today [2 marks] Find the coordinates of the points where $y = x^2 - 3$ meets $y = 2x$.

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Sketch $y = -\frac{3}{x}$, stating the equations of its asymptotes.

Hint $y = \frac{k}{x}$ with $k < 0$: when $x > 0$, $y < 0$, and when $x < 0$, $y > 0$.

B1 Sketch $y = \frac{5}{x^2}$, stating the equations of its asymptotes. Write down the set of values that y takes. [3 marks]

EXAMPLE 2

On the same axes sketch $y = \frac{2}{x}$ and $y = \frac{8}{x}$. Find the set of values of x for which $\frac{8}{x} > \frac{2}{x}$.

Hint For $x > 0$ the bigger numerator gives the bigger value. For $x < 0$ both are negative.

B2 On the same axes sketch $y = -\frac{2}{x^2}$ and $y = -\frac{7}{x^2}$. Find the set of values of x for which $-\frac{7}{x^2} < -\frac{2}{x^2}$. [4 marks]

Answer

EXAMPLE 3

On the same axes sketch

$y = x^2(x + 3)$ and $y = -\frac{2}{x^2}$. Hence state, with a reason, the number of real solutions of $x^4(x + 3) + 2 = 0$.

Hint Divide $x^4(x + 3) + 2 = 0$ by x^2 : it says $x^2(x + 3) = -\frac{2}{x^2}$, so count where the curves meet.

B3

On the same axes

[4 marks]

sketch

$y = (x + 2)^2(x - 1)$ and

$y = \frac{4}{x}$, showing where

the cubic meets the

axes. Hence state, with

a reason, the number of

real solutions of

$x(x + 2)^2(x - 1) = 4$.

Answer

EXAMPLE 4

Find the coordinates of the points

where $y = \frac{10}{x}$ meets $y = 2x + 1$.

Hint Set $\frac{10}{x} = 2x + 1$ and multiply by x .

B4

Find the coordinates of [5 marks]

the points where

$y = \frac{15}{x}$ meets

$y = 3x - 4$.

Answer

- C1** On the same axes sketch $y = \frac{3}{x}$ and $y = x^2 - 4$. Hence state, giving [4 marks]
a reason, the number of real solutions of $x^3 - 4x - 3 = 0$.

Answer

- C2** a and b are positive constants. On the same axes sketch $y = -\frac{a}{x}$ and [4 marks]
 $y = x(x + b)^2$, showing where each curve meets the axes. Explain
why $x^2(x + b)^2 + a = 0$ has no real solutions.

- C3** On the same axes sketch $y = x^3 + x^2 - 9x$ and $y = 3x$. Find the [5 marks]
coordinates of the points where they meet.

Answer

D1 The curve C has equation $y = \frac{9}{x^2}$ and the line L has equation $y = 4x + 13$.

- (a) On the same axes sketch C and L , showing where L meets the axes. [3 marks]

- (b) Using your sketch, state the number of points where C and L meet. [1 mark]

Answer

- (c) Show that the x -coordinates of these points satisfy [2 marks]
 $(x + 1)(x + 3)(4x - 3) = 0$.

- (d) Hence find the exact coordinates of the points where C and L meet. [3 marks]

Answer

Total for D1: 9 marks

D2

Ravi sketches $y = x(x - 5)$ and $y = -\frac{4}{x}$. He says: "For $x < 0$ the curves never meet, so $x(x - 5) = -\frac{4}{x}$ has exactly one real solution."

Ravi is wrong.

- (a) Show that the equation can be written as $x^3 - 5x^2 + 4 = 0$, and [2 marks]
verify that $x = 1$ is a solution.

- (b) Hence find all the solutions exactly, and say what Ravi got wrong. [3 marks]

Answer

Total for D2: 5 marks

- D3** The curve C has equation $y = \frac{4}{x}$ and the line L has equation [5 marks]
 $y = k - x$, where k is a constant. Find the set of values of k for which
 L meets C at two distinct points.

Answer

D4 The curve C has equation $y = \frac{12}{x}$ and the curve D has equation $y = x^2 + x$.

(a) On the same axes sketch C and D , showing where D meets the x -axis. [2 marks]

(b) Show that the x -coordinates of the points where C and D meet satisfy $x^3 + x^2 - 12 = 0$, and that $x = 2$ is one of them. [2 marks]

(c) Hence show that C and D meet at exactly one point, and find its coordinates. [4 marks]

Answer

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The curve $y = \frac{k}{x^2}$, where k is a non-zero constant, and the line $y = 3 - x$ can meet at one, two or three points. For each of these numbers, find the set of values of k that gives it. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** They meet where $k = x^2(3 - x)$, $x \neq 0$. So count where the line $y = k$ meets $y = x^2(3 - x)$.
- Hint 2** $y = k$ touches the cubic where $x^3 - 3x^2 + k$ has a repeated root. Write it as $(x - a)^2(x - b)$ and compare coefficients.
- Hint 3** $2a + b = 3$ and $a^2 + 2ab = 0$, so $a = 2$, $b = -1$ and $k = 4$: the cubic touches the axis at 0 and has its top at $(2, 4)$. For $0 < k < 4$, $y = k$ meets it once for $x < 0$ and twice for $0 < x < 3$: three points. For $k = 4$: at $x = 2$ and once for $x < 0$, two points. For $k > 4$ only $x < 0$, and for $k < 0$ only $x > 3$: one point.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch $y = k/x$ and $y = k/x^2$. ☐ ☐ ☐

I can count solutions from a sketch. ☐ ☐ ☐

I can find where two curves meet. ☐ ☐ ☐

I can find when a line meets a curve twice. ☐ ☐ ☐