Identities with sec, cosec and cot

117 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-reciprocal-trig-identities/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Show that $\dfrac{1-\cos^2 x}{\sin x \cos x} \equiv \tan x$.

Answer .....

A2 Needed today [3 marks]

Solve $2\sin^2 x - \sin x - 1 = 0$ for $0° \leq x < 360°$.

Answer .....

A3 Today's identity [2 marks]

Divide every term of $\sin^2 x + \cos^2 x \equiv 1$ by $\cos^2 x$. What identity do you get?

Answer .....

A4 Last lesson [3 marks]

Find the exact value of $\operatorname{cosec}\left(-\dfrac{2\pi}{3}\right) + \cot\dfrac{\pi}{4}$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Prove that $(\operatorname{cosec} x - \sin x)\tan x \equiv \cos x$.

Hint Write everything in sin and cos, then use 1 – sin² x = cos² x.

B1 Prove that $(\sec x - \cos x)\cot x \equiv \sin x$. [3 marks]

EXAMPLE 2

Prove that $\dfrac{1+\tan^2 x}{1+\cot^2 x} \equiv \tan^2 x$.

Hint Use the two new identities on the top and on the bottom first.

B2 Prove that $\sec^4 x - \tan^4 x \equiv \sec^2 x + \tan^2 x$. [3 marks]

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EXAMPLE 3

Solve $2\tan^2 x + \sec x = 1$ for $0° \leq x < 360°$. Give your answers to 1 decimal place.

Hint Replace tan² x by sec² x – 1 so the equation is a quadratic in sec x.

B3 Solve $2\cot^2 x - \operatorname{cosec} x - 4 = 0$ for $0° < x < 360°$. Give your answers to 1 decimal place. [6 marks]

Answer .....

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EXAMPLE 4

Solve $\cot\left(2x + \frac{\pi}{3}\right) = 1$ for $0 \leq x \leq \pi$. Give your answers in terms of π.

Hint Let u = 2x + π/3 and find the interval u lives in before you solve tan u = 1.

B4 Solve $\operatorname{cosec}\left(2x - \frac{\pi}{4}\right) = \sqrt{2}$ for $0 \leq x \leq \pi$. Give your answers in terms of π. [5 marks]

Answer .....

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EXAMPLE 5

Given that $\operatorname{cosec} A = -\sqrt{5}$ and $\frac{3\pi}{2} < A < 2\pi$, find the exact values of $\cot A$ and $\sec A$.

Hint Use cosec² A = 1 + cot² A, then choose the sign that fits the quadrant.

B5 Given that $\sec B = -\sqrt{10}$ and $\frac{\pi}{2} < B < \pi$, find the exact values of $\tan B$ and $\operatorname{cosec} B$. [4 marks]

Answer .....

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C1 Ben solves $\tan x \sec x = 2 \tan x$ for $0 \leq x \leq 2\pi$. He divides both sides by $\tan x$, gets $\sec x = 2$, and gives the answers $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$. Explain why Ben's answer is not complete, and give all the solutions. [5 marks]

Answer .....

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C2 Leah solves $\tan^2 x = 2\sec x + 2$ for $0° \leq x < 360°$. She writes $\tan^2 x = 1 - \sec^2 x$ and finds only $x = 180°$. Find Leah's mistake and solve the equation correctly, giving your answers to 1 decimal place. [5 marks]

Answer .....

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C3 Solve $2\cot^2 x = 7\operatorname{cosec} x + 2$ for $-180° \leq x \leq 180°$. Give your answers to 1 decimal place. [5 marks]

Answer .....

C4 Find a Cartesian equation for each curve.

(a) $x = 3\sec t$, $y = 2\tan t$ [3 marks]

Answer .....

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(b) $x = 1 + \cot t,\ y = \operatorname{cosec} t$ [2 marks]

Answer .....

Total for C4: 5 marks

C5 Prove that $\dfrac{1 + \sec x}{\tan x + \sin x} \equiv \operatorname{cosec} x.$ [4 marks]

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D1 This question is about the equation $\tan^2\theta = 3\sec\theta - 3$.

(a) Show that the equation can be written as $\sec^2\theta - 3\sec\theta + 2 = 0$. [2 marks]

(b) Hence solve $\tan^2\theta = 3\sec\theta - 3$ for $0° \leq \theta < 360°$. [4 marks]

Answer .....

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(c) Hence solve $\tan^2(2x - 30^\circ) = 3\sec(2x - 30^\circ) - 3$ for $0^\circ \leq x < 360^\circ$. [4 marks]

Answer .....

Total for D1: 10 marks

D2

Mia is told that $\cot\theta = 2\sqrt{2}$ and $\pi < \theta < \frac{3\pi}{2}$. She writes: "$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 9$, so $\operatorname{cosec}\theta = 3$."

Mia is wrong.

(a) Explain Mia's mistake and give the correct value of cosec θ. [2 marks]

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(b) Find the exact value of $\sec\theta$, giving your answer with a rational denominator. [3 marks]

Answer .....

(c) Show that $\cot\theta + \tan\theta \equiv \operatorname{cosec}\theta\sec\theta$. Hence find the exact value of $\cot\theta + \tan\theta$, in the form $\dfrac{a\sqrt{2}}{b}$. [4 marks]

Answer .....

Total for D2: 9 marks

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D3 Solve $\operatorname{cosec}^2 x + \cot x = 3$ for $0 \leq x < 2\pi$. Give exact answers where you can, and the others to 3 significant figures. [7 marks]

Answer .....

D4

(a) Prove that $(1 - \cos x)(1 + \sec x) \equiv \sin x \tan x$. [3 marks]

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(b) Hence solve $(1 - \cos x)(1 + \sec x) = \frac{3}{2}$ for $0 \leq x \leq 2\pi$, giving your answers in terms of π. [6 marks]

Answer .....

Total for D4: 9 marks

D5 A curve C has parametric equations $x = 2 + \sec t$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$.

(a) Show that a Cartesian equation of C is $(x - 2)^2 - y^2 = 1$. [3 marks]

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(b) State the range of values that x takes on C, giving a reason. [2 marks]

Answer .....

(c) Hence find the coordinates of the points where C meets the line $y = 2x - 6$. [5 marks]

Answer .....

Total for D5: 10 marks

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D6

(a) Show that $\sec^2\theta + \operatorname{cosec}^2\theta \equiv \sec^2\theta \operatorname{cosec}^2\theta$. [3 marks]

(b) Hence solve $\sec^2\theta + \operatorname{cosec}^2\theta = \dfrac{16}{3}$ for $0^\circ < \theta < 180^\circ$. [5 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Solve $\sec x + \tan x = 2$ for $0 \leq x < 2\pi$, giving your answer to 3 significant figures. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the equation in $\sin x$ and $\cos x$ and multiply by $\cos x$: $1 + \sin x = 2\cos x$.

Hint 2 Square both sides and use $\cos^2 x = 1 - \sin^2 x$ to get a quadratic in $\sin x$.

Hint 3 Squaring can add roots. Put each value back into $\sec x + \tan x = 2$, and remember $\cos x$ cannot be 0.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can derive 1 + tan²x ≡ sec²x and 1 + cot²x ≡ cosec²x from sin²x + cos²x ≡ 1. ○ ○ ○

I can prove an identity by writing everything in terms of sin x and cos x. ○ ○ ○

I can solve a quadratic equation in sec, cosec or cot, rejecting values that are impossible. ○ ○ ○

I can find sec, cosec and cot of an angle from one of them and the quadrant. ○ ○ ○

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