Secant, cosecant and cotangent and their graphs

117 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-sec-cosec-cot-graphs/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Write down the exact values of $\sin\frac{\pi}{3}$, $\cos\frac{3\pi}{4}$ and $\tan\frac{5\pi}{6}$.

Answer .....

A2 Needed today [2 marks]

Write down the period, in radians, of $y = \cos 2x$ and of $y = \tan\frac{x}{2}$.

Answer .....

A3 Common slip [2 marks]

Sam writes $\sec 60° = \frac{1}{\sin 60°}$. Give the correct definition of $\sec x$ and find the exact value of $\sec 60°$.

Answer .....

A4 Last chapter [3 marks]

Describe the two transformations that map $y = \cos x$ onto $y = 3\cos x - 1$, and state the range of $y = 3\cos x - 1$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the exact values of $\sec 135°$, $\operatorname{cosec}\left(-\frac{\pi}{3}\right)$ and $\cot \frac{4\pi}{3}$.

Hint Find sin, cos or tan of the angle first, with its sign. Then take the reciprocal.

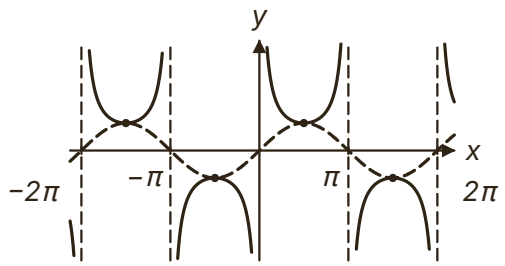
B1 Find the exact values of $\sec\left(-\frac{5\pi}{6}\right)$, $\operatorname{cosec} 210°$ and $\cot \frac{11\pi}{6}$. [4 marks]

Answer .....

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EXAMPLE 2

Sketch $y = \operatorname{cosec} x$ for $-2\pi \leq x \leq 2\pi$. Give the equations of the asymptotes, the coordinates of the turning points and the range.



Hint Draw y = sin x lightly first. Where sin x = 0 there is an asymptote; where sin x = ±1 there is a turning point.

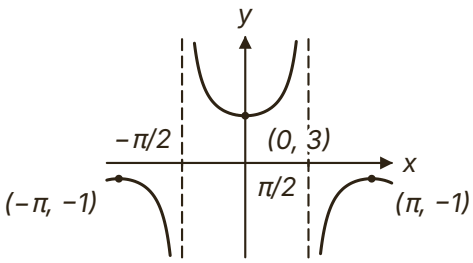
B2 Sketch $y = \sec x$ for $-\frac{5\pi}{4} \leq x \leq \frac{5\pi}{4}$. Give the equations of the asymptotes, the coordinates of the turning points and the range. [5 marks]

Answer .....

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EXAMPLE 3

Sketch $y = 1 + 2\sec x$ for $-\pi \leq x \leq \pi$. Give the asymptotes, the turning points and the range.



Hint Stretch y = sec x by 2 vertically, then move it up 1. The asymptotes do not move.

B3 Sketch $y = 4 - 3\,\mathrm{cosec}\,2x$ for $0 < x < \pi$. Give the equation of the asymptote, the coordinates of the turning points, saying which is a maximum and which a minimum, and the range. [6 marks]

Answer .....

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EXAMPLE 4

Explain why $\sec x = \frac{1}{2}$ has no solutions. For which values of k does $\operatorname{cosec} x = k$ have exactly two solutions with $0 < x < 2\pi$?

Hint Look at the range of each graph and draw a horizontal line y = k across it.

B4 Find the values of k for which the equation $2\sec x - 1 = k$ has exactly one solution with $-\frac{\pi}{2} < x < \frac{3\pi}{2}$, $x \neq \frac{\pi}{2}$. [5 marks]

Answer .....

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EXAMPLE 5

Show that $\cot x = -\tan\left(x - \frac{\pi}{2}\right)$.

Hence describe how the graph of $y = \cot x$ can be obtained from the graph of $y = \tan x$.

Hint Write tan as sin over cos, and use sin(x − π/2) = −cos x and cos(x − π/2) = sin x.

B5 Show that $\sec x = \operatorname{cosec}\left(x + \frac{\pi}{2}\right)$. [4 marks]

Hence describe the single transformation that maps $y = \operatorname{cosec} x$ onto $y = \sec x$.

Answer .....

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C1 Jo works out $\operatorname{cosec}\frac{5\pi}{3} - \sec\frac{5\pi}{3}$ and gets $2 + \frac{2\sqrt{3}}{3}$. Explain the mistake Jo makes and find the correct exact value. [4 marks]

Answer .....

C2 The function f is defined by $\mathrm{f}(x) = 5 - 2\operatorname{cosec} x$, $0 < x < 2\pi$, $x \neq \pi$. A student says that the range of f is $3 \leq \mathrm{f}(x) \leq 7$. Explain why the student is wrong and find the range of f. [5 marks]

Answer .....

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C3 For the curve $y = \operatorname{cosec}\left(\frac{x}{2} + \frac{\pi}{6}\right)$, $-2\pi \leq x \leq 2\pi$, find the equations of the asymptotes and the coordinates of the turning points, stating which is a maximum and which is a minimum. [6 marks]

Answer .....

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C4 The curve $y = a + b\sec x$, $-\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$, $x \neq \dfrac{\pi}{2}$, where a and b are constants and $b > 0$, has a minimum point at $(0, 7)$ and a maximum point at $(\pi, -1)$. Find a and b, and state the range of the curve. [5 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1 The function f is defined by $\mathrm{f}(x) = 3\sec 2x + 1$, $0 \leq x \leq \pi$, $x \neq \frac{\pi}{4}$, $x \neq \frac{3\pi}{4}$.

(a) Show that the graph of $y = \mathrm{f}(x)$ has asymptotes at $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$, and no others. [2 marks]

Answer .....

(b) Find the coordinates of the points on the graph where the gradient is zero. [3 marks]

Answer .....

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(c) State the range of f. [2 marks]

Answer .....

(d) Hence find the set of values of k for which the equation $\mathrm{f}(x) = k$ has exactly two solutions. [3 marks]

Answer .....

Total for D1: 10 marks

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D2

Ravi is finding the asymptotes of $y = 2\,\text{cosec}\left(x - \frac{\pi}{4}\right)$, $0 \leq x \leq 2\pi$. He writes: "The graph of $y = 2\,\text{cosec}\,x$ has asymptotes at $x = \pi$ and $x = 2\pi$. Moving it by $\frac{\pi}{4}$ gives asymptotes at $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$."

Ravi is wrong.

(a) Explain Ravi's mistake and give the correct asymptotes. [3 marks]

(b) Solve $2\,\text{cosec}\left(x - \frac{\pi}{4}\right) = -2\sqrt{2}$ for $0 \leq x \leq 2\pi$. [4 marks]

Answer .....

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(c) State the range of the curve. [2 marks]

Answer .....

Total for D2: 9 marks

D3 Find the coordinates of every point where the curve $y = \sec x$ meets the curve $y = 2\cos x$, for $0 \leq x \leq 2\pi$. Give your answers exactly. [7 marks]

Answer .....

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D4

(a) Show that $\cot x = \tan\left(\frac{\pi}{2} - x\right)$. [2 marks]

(b) Using part (a), describe two transformations that map $y = \tan x$ onto $y = \cot x$. [2 marks]

Answer .....

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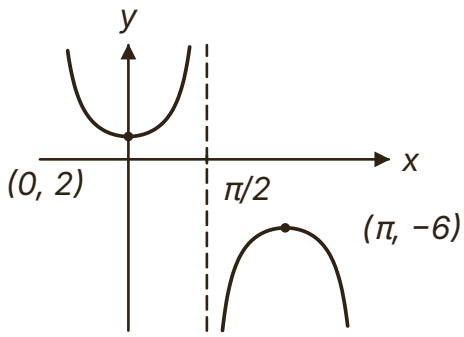
(c) Hence solve $\cot 2x = \tan x$ for $0 < x < \pi$, giving your answers in terms of π. [5 marks]

Answer .....

Total for D4: 9 marks

D5 The figure shows part of the curve with equation $y = p + q\sec x$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$, $x \neq \frac{\pi}{2}$, where p and q are constants. The curve has a minimum point at $(0, 2)$ and a maximum point at $(\pi, -6)$.

(a) Find the value of p and the value of q. [3 marks]



Answer .....

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(b) State the range of the curve. [2 marks]

Answer .....

(c) Hence find the exact values of x where the curve meets the line $y = -10$. [4 marks]

Answer .....

(d) Explain why the curve does not meet the x-axis. [2 marks]

Total for D5: 11 marks

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D6 Given that $\sec\theta = -\frac{5}{4}$ and $\pi < \theta < \frac{3\pi}{2}$,

(a) find the exact value of $\operatorname{cosec}\theta$, [3 marks]

Answer .....

(b) find the exact value of $\cot\theta$, [2 marks]

Answer .....

(c) hence find the exact value of $\operatorname{cosec}\theta - \cot\theta$. [2 marks]

Answer .....

Total for D6: 7 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The function f is defined by $\mathrm{f}(x) = a + b\,\mathrm{cosec}(x + c)$, $0 \leq x \leq 2\pi$, where a, b and c are constants, $b > 0$ and $0 < c < \pi$. The graph of $y = \mathrm{f}(x)$ has an asymptote at $x = \dfrac{2\pi}{3}$, and the range of f is $\mathrm{f}(x) \leq -1$ or $\mathrm{f}(x) \geq 5$. Find a, b and c, and solve $\mathrm{f}(x) = 2 + 2\sqrt{3}$. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 An asymptote is where $\sin(x + c) = 0$. Which multiple of π can $\frac{2\pi}{3} + c$ be, when $0 < c < \pi$?

Hint 2 The two ends of the range come from $\text{cosec} = 1$ and $\text{cosec} = -1$: so $a + b = 5$ and $a - b = -1$. Check both values are reached on the domain.

Hint 3 For the equation you need $\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ with $\frac{\pi}{3} \leq x + \frac{\pi}{3} \leq \frac{7\pi}{3}$. Look at both ends of the interval.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can find exact values of sec, cosec and cot for angles in degrees or radians. ○ ○ ○
- I can sketch y = sec x, y = cosec x and y = cot x and give their asymptotes and turning points. ○ ○ ○
- I can sketch a transformed graph such as y = a + b sec(kx) and state its range. ○ ○ ○
- I can use a graph to say how many solutions an equation has. ○ ○ ○

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