Sigma notation and recurrence relations

115 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-sigma-notation-and-recurrence-relations/

Date

A Do Now

Quick questions to start. They use what you need today.

A1 Needed today [2 marks]

Evaluate $\sum_{r=1}^{4}(2r+1)$.

Answer .....

A2 Common slip [2 marks]

How many terms are there in $\sum_{r=5}^{30} r^2$?

Answer .....

A3 Needed today [2 marks]

A sequence has $u_1 = 4$ and $u_{n+1} = 3u_n - 2$. Find u_3.

Answer .....

A4 From chapter 3 [2 marks]

Find the sum of the first 6 terms of the geometric series $5 + 15 + 45 + \cdots$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Evaluate $\sum_{r=1}^{30}(4r-1)$.

Hint Find the first term, the last term and the number of terms. It is an arithmetic series.

B1 [4 marks]

Evaluate $\sum_{r=10}^{50}(100-2r)$.

Answer .....

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EXAMPLE 2

Evaluate $\sum_{r=1}^{8} 3 \times 2^r$.

Hint Write out the first term. When $r = 1$ the term is 6, not 3.

B2 [4 marks]

Evaluate $\sum_{r=0}^{9} 5(0.5)^r$, giving your answer as an exact fraction.

Answer .....

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EXAMPLE 3

Evaluate $\sum_{r=1}^{20}(2^r + 3r)$.

Hint Split it into $\sum 2^r$ (geometric) and $\sum 3r$ (arithmetic). Find each, then add.

B3 [5 marks]

Evaluate $\sum_{r=1}^{10}(3^r - 4r + 1)$.

Answer .....

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EXAMPLE 4

A sequence has $u_1 = 2$ and $u_{n+1} = ku_n + 3$. Given $u_3 = 23$, find the possible values of k.

Hint Write u_2 and then u_3 in terms of k. Set $u_3 = 23$ and solve the quadratic.

B4 A sequence has $u_1 = 3$ and $u_{n+1} = ku_n - 4$. Given $u_3 = 11$, find the possible values of k. [5 marks]

Answer .....

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EXAMPLE 5

A sequence has $u_1 = 1$ and $u_{n+1} = 4 - u_n$. Find $\sum_{r=1}^{25} u_r$.

Hint Write out the first few terms. When a term repeats, the sequence repeats. Count the full cycles in 25 terms.

B5 [5 marks]

A sequence has $u_1 = 2$ and $u_{n+1} = \dfrac{1}{1 - u_n}$.

Find $\sum_{r=1}^{50} u_r$.

Answer .....

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C1 A sequence has $u_1 = 1$ and $u_{n+1} = u_n + \dfrac{1}{u_n}$. Find u_2, u_3 and u_4, and prove that the sequence is increasing. [4 marks]

Answer $u_2 = \ldots$, $u_3 = \ldots$, $u_4 = \ldots$

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C2 [5 marks]

Show that $\sum_{r=1}^{n}(6r-1) = 3n^2 + 2n$, and find the least n for which the sum is greater than 2000.

Answer .....

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C3 A sequence has $u_1 = 2$ and $u_{n+1} = 0.5u_n + 6$. Find u_2, u_3 and u_4. The sequence tends to a limit L. Find L. [5 marks]

Answer .....

C4 A sequence has $u_1 = 5$ and $u_{n+1} = 3u_n - 8$. A student writes: "$L = 3L - 8$ gives $L = 4$, so the sequence tends to 4." Find u_2, u_3 and u_4, and explain why the student is wrong. [4 marks]

Answer $u_2 = \ldots$, $u_3 = \ldots$, $u_4 = \ldots$

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C5 Find $\sum_{r=1}^{\infty} 8\left(-\frac{1}{2}\right)^r$. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 A sequence is given by $u_1 = 2$, $u_{n+1} = au_n + b$, where a and b are constants. Given $u_2 = 7$ and $u_3 = 22$:

(a) find a and b. [3 marks]

Answer .....

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(b) Find u_5. [2 marks]

Answer .....

(c) Find $\sum_{r=1}^{5} u_r$. [2 marks]

Answer .....

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(d) Show that $u_n = \dfrac{5 \times 3^{n-1} - 1}{2}$ gives $u_1 = 2$ and satisfies $u_{n+1} = 3u_n + 1$. [3 marks]

Answer .....

Total for D1: 10 marks

D2

Zoe says: "$\sum_{r=4}^{12}(3r - 1)$ has 8 terms, starting at 11 and going up by 3, so it equals $\dfrac{8}{2}(22 + 21) = 172$."

Zoe is wrong.

(a) Find the correct value of the sum, and explain the mistake Zoe made. [3 marks]

Answer .....

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(b) Given that $\sum_{r=4}^{n}(3r-1)=1350$, find n. [5 marks]

Answer .....

Total for D2: 8 marks

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D3 [7 marks]

Given that $\sum_{r=1}^{n}(2r+k) = n^2 + 7n$ for all positive integers n, find the constant k, and hence find $\sum_{r=11}^{20}(2r+k)$.

Answer .....

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D4 A sequence has $u_1 = 4$ and $u_{n+1} = \dfrac{2u_n + k}{u_n - 2}$, where k is a constant and $k \neq -4$.

(a) Find u_2 in terms of k. [2 marks]

Answer .....

(b) Show that $u_3 = 4$. [3 marks]

Answer .....

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(c) Given $\sum_{r=1}^{30} u_r = 225$, find k. [3 marks]

Answer .....

(d) Find the value of k for which every term of the sequence is 4. [2 marks]

Answer .....

Total for D4: 10 marks

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D5 The sum $S_n = \sum_{r=1}^{n} 5 \times 2^r$.

(a) Show that $S_n = 10(2^n - 1)$. [3 marks]

Answer .....

(b) Find the least n for which $S_n > 10^6$. [3 marks]

Answer .....

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(c) Given $\sum_{r=n+1}^{2n} 5 \times 2^r = 652\,800$, find n. [3 marks]

Answer .....

Total for D5: 9 marks

D6 A sequence has $u_1 = 3$ and $u_{n+1} = ku_n + 5$, where k is a constant.

(a) Show that $\sum_{r=1}^{3} u_r = 3k^2 + 8k + 13$. [3 marks]

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(b) Given that $\sum_{r=1}^{3} u_r = 16$, find the two possible values of k. [3 marks]

Answer .....

(c) Hence find the value of k for which the sequence converges, and find its limit. Explain why the sequence does not converge for the other value. [3 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A sequence has $u_{n+1} = u_n^2 - 6$. Given that $u_3 = u_1$ but $u_2 \neq u_1$ [9 marks]

, find the possible values of u_1 in exact form, and find $\sum_{r=1}^{100} u_r$.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $u_1 = x$. Then $u_3 = (x^2 - 6)^2 - 6 = x$, a quartic.

Hint 2 The values with $u_2 = u_1$ solve $x^2 - 6 = x$, so $x^2 - x - 6$ is a factor of the quartic.

Hint 3 The other factor is $x^2 + x - 5$. Use it to write $u_1 + u_2$ simply.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can count the terms in $\sum_{r=a}^{b}$ and evaluate arithmetic and geometric sums written in sigma notation. ○ ○ ○

I can split a sum into parts and find a sum that does not start at $r = 1$. ○ ○ ○

I can use a recurrence relation to find terms and unknown constants. ○ ○ ○

I can spot a periodic sequence and use it to find a long sum. ○ ○ ○

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