

Simultaneous equations, linear and quadratic

72 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-simultaneous-equations-linear-and-quadratic/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|---|
| <p>A1 Needed today [2 marks] Solve $2x + y = 13$ and $x - 3y = -4$.</p> | <p>A2 Needed today [2 marks] Expand and simplify $(5 - 2y)^2$.</p> |
|---|---|

Answer $x = \dots\dots\dots$ $y = \dots\dots\dots$

Answer

- | | |
|---|---|
| <p>A3 Last lesson [2 marks] Write $3 + 8x - 2x^2$ in the form $q - p(x - r)^2$.</p> | <p>A4 From chapter 2 [2 marks] Work out the discriminant of $2x^2 - 7x + 8$. How many real roots does $2x^2 - 7x + 8 = 0$ have?</p> |
|---|---|

Answer

Answer

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve the simultaneous equations

$$3x + 4y = 5 \text{ and } 5x - 6y = 21.$$

Hint Make the y -terms the same size: multiply the first by 3 and the second by 2, then add.

B1 Solve the simultaneous equations $4x - 3y = 23$ and $6x + 5y = 6$. [3 marks]

Answer $x = \dots\dots$ $y = \dots\dots$

EXAMPLE 2

Solve the simultaneous equations

$$x + 3y = -2 \text{ and } x^2 + xy = 8.$$

Hint Write $x = -2 - 3y$ and put it, in brackets, into the second equation. Find each y , then the x that goes with it.

B2 Solve the simultaneous equations $y = x + 2$ and $x^2 + 3xy - y^2 = 4$. [5 marks]

Answer $x = \dots\dots, y = \dots\dots$ and $x = \dots\dots, y = \dots\dots$

EXAMPLE 3

The line $y = 3x + k$ meets the curve $y = x^2 - x + 2$ at exactly one point. Find k , and the coordinates of that point.

Hint Put the two expressions for y equal. One meeting point means the quadratic has one root, so its discriminant is 0.

B3 The line $y = k - 3x$, [4 marks]
where k is a positive constant, meets the curve $xy = 27$ at exactly one point. Find k , and the coordinates of that point.

Answer $k = \dots\dots\dots$ $(\dots\dots, \dots\dots)$

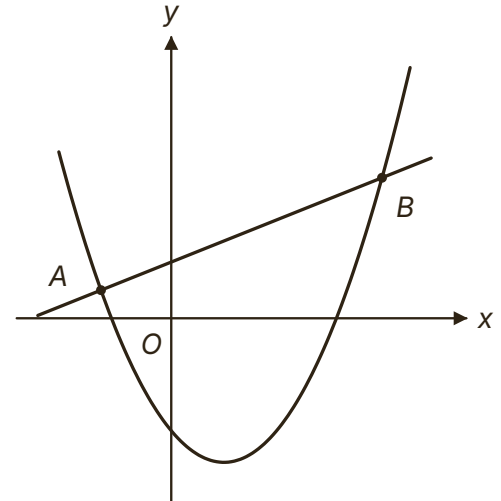
c Practice

A-Level

C1 Solve the simultaneous equations $3(x + 2y) = 2x - 1$ and [4 marks]
 $\frac{x - y}{2} = y + 4.$

Answer $x = \dots\dots\dots$ $y = \dots\dots\dots$

- C2** The sketch shows the curve $y = 2x^2 - 3x - 4$ and the line $y = x + 2$. [4 marks]
They meet at A and B . Find the coordinates of A and B .



Answer $A(\dots, \dots)$ $B(\dots, \dots)$

Not drawn accurately

- C3** The simultaneous equations $y = kx - 1$ and $x^2 + xy = 10$, where k is a constant, have one solution $x = 2$, $y = p$. Find p and k , and find the other solution. [5 marks]

Answer $p = \dots$ $k = \dots$ $x = \dots, y = \dots$

D1

Ella writes: $x^2 - x \times 3 - 2x = 18$, so $x^2 - 5x - 18 = 0$, and the answers are not whole numbers.

Ella is solving the simultaneous equations $y = 3 - 2x$ and $x^2 - xy = 18$.

(a) Explain the mistake Ella has made. [2 marks]

(b) Show that $x^2 - x - 6 = 0$. [2 marks]

(c) Hence solve the simultaneous equations. [4 marks]

Answer $x = \dots\dots, y = \dots\dots$ and $x = \dots\dots, y = \dots\dots$

Total for D1: 8 marks

D2 Here are two simultaneous equations: $y = 8 - 3x$ and $x^2 + xy + 6 = 0$.

(a) By eliminating y , show that $x^2 - 4x - 3 = 0$. [3 marks]

- (b) Hence solve the simultaneous equations. Give your answers in the form $a + b\sqrt{7}$, where a and b are integers. [5 marks]

Answer $x = \dots\dots, y = \dots\dots$ and $x = \dots\dots, y = \dots\dots$

Total for D2: 8 marks

- D3** $kx + 2y = 8$ and $3x - y = 1$ are simultaneous equations, where k is a constant.

- (a) Find x and y in terms of k . [3 marks]

Answer $x = \dots\dots$ $y = \dots\dots$

- (b) Explain what happens to the two lines when $k = -6$. [2 marks]

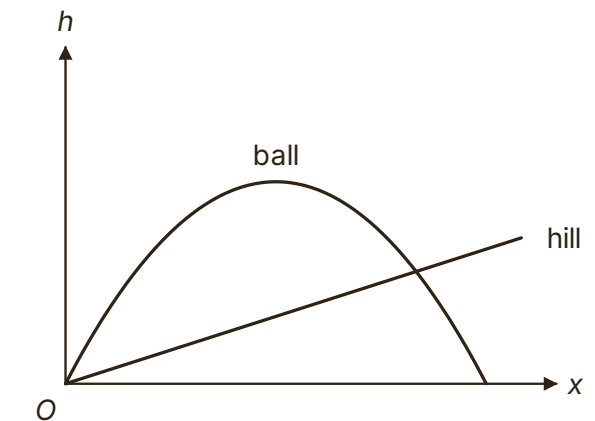
- (c) The two lines meet on the x -axis. Hence find k . [2 marks]

Answer $k = \dots\dots$

Total for D3: 7 marks

D4 A golf ball is hit from a point O at the foot of a hill. The height of the ball, h m, at a horizontal distance x m from O is modelled by $h = 1.2x - 0.05x^2$. The surface of the hill is modelled by $h = 0.2x$.

- (a) Find how far from O , [4 marks]
horizontally, the ball
lands on the hill, and how
high it lands.



Not drawn accurately

Answer m, height m

- (b) Hence find the greatest vertical height of the ball above the hill, [3 marks]
and the value of x where it happens.

Answer m $x = \dots\dots$

- (c) Find the two horizontal distances from O at which the height of [3 marks]
the ball above the hill is 3.2 m.

Answer m and m

Total for D4: 10 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Solve the simultaneous equations $x^2 + xy = 12$ and $y^2 + xy = 24$. [6 marks]

Answer $x = \dots\dots, y = \dots\dots$ and $x = \dots\dots, y = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Add the two equations. Look at the left-hand side.
- Hint 2** $(x + y)^2 = 36$, so $x + y = 6$ or -6 . The first equation is $x(x + y) = 12$.
- Hint 3** If $x + y = 6$ then $6x = 12$, so $x = 2$, $y = 4$. Now try $x + y = -6$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve linear simultaneous equations. ☐ ☐ ☐

I can solve a linear and a quadratic pair. ☐ ☐ ☐

I can pair each x with its own y . ☐ ☐ ☐

I can find k when a line touches a curve. ☐ ☐ ☐