Sketching curves and points of intersection

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-sketching-parametric-curves-intersections/

Date

A Do Now

Quick questions to start. Some check what you need today; some are slips that often cost marks.

A1 Needed today [2 marks]

Find the coordinates of the points where the line $y = 2x + 1$ meets the circle $x^2 + y^2 = 13$.

Answer .....

A2 Needed today [2 marks]

Solve $\sin t + \cos t = 1$ for $0 \leq t < 2\pi$.

Answer .....

A3 Last lesson [2 marks]

A curve has parametric equations $x = t^2 + 1$, $y = 2t$. Find the value of t at the point $(5, -4)$.

Answer .....

A4 Common slip [2 marks]

A curve has parametric equations $x = \cos t$, $y = \sin t$, $0 \leq t \leq \pi$. Describe the curve.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Sketch the curve with parametric equations $x = t - 1$, $y = t^2 - 4$, $-1 \leq t \leq 3$. Mark the end points and where it meets the axes.

Hint Find the points at the two ends of the interval. Then set x = 0 and y = 0, and keep only values of t in the interval.

B1 Sketch the curve with parametric equations $x = t + 2$, $y = t^2 - 1$, $-2 \leq t \leq 2$. Mark the end points, the lowest point and where it meets the axes. [5 marks]

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EXAMPLE 2

A curve has parametric equations $x = t^2 - 4$, $y = t^3 - t$. Find the points where it meets the coordinate axes.

Hint Set y = 0 and solve for every t, then find x. Do the same with x = 0.

B2 A curve has parametric equations $x = t^2 - 1$, $y = t^3 - 4t$. Find the coordinates of the points where it meets the coordinate axes. [4 marks]

Answer .....

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EXAMPLE 3

A curve has parametric equations $x = 2t^2 - 1$, $y = 3t$. Find where it meets the line $x - y = 4$.

Hint Put the parametric x and y into the equation of the line. Solve for t, then find the points.

B3 A curve has parametric equations $x = t^2$, $y = 2t$. Find the coordinates of the points where it meets the line $y = x - 3$. [4 marks]

Answer .....

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EXAMPLE 4

A curve has parametric equations $x = 2\sin t$, $y = 3\cos 2t$, $0 \leq t < 2\pi$. Find the points where it meets the line $y = \frac{3}{2}$.

Hint Solve cos 2t = 1/2 for 2t in the interval 0 ≤ 2t < 4π, then halve.

B4 [5 marks]

A curve has parametric equations $x = 3\cos t$, $y = \cos 2t + 1$, $0 \leq t < 2\pi$. Find the values of t, and the points, where it meets the line $y = \frac{1}{2}$.

Answer .....

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EXAMPLE 5

A curve has parametric equations $x = t + k$, $y = kt^2$, where k is a constant. The curve passes through $(3, 4)$. Find the possible values of k.

Hint Make t the subject of the x equation at the point, then put it into the y equation.

B5 A curve has parametric equations $x = t - k$, $y = kt$, where k is a constant. The curve passes through $(1, 6)$. Find the possible values of k, and the value of t at the point for each. [4 marks]

Answer .....

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C1 A curve has parametric equations $x = 3t$, $y = t^2$. Find the coordinates of the points where it meets the line $y = x - 2$. [4 marks]

Answer .....

C2 A curve has parametric equations $x = 2\cos t$, $y = 2\sin t$, $\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$. Lee says the curve is a circle. Describe the curve correctly and give its end points. [4 marks]

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C3 A curve has parametric equations $x = t^2$, $y = t^3 - 4t$. Kai divides $t^3 - 4t = 0$ by t, gets $t = \pm 2$, and says the curve meets the x-axis only at $(4, 0)$. Find every point where the curve meets the coordinate axes. [4 marks]

Answer .....

C4 A curve has parametric equations $x = t^2 - 2t$, $y = t^2 + t - 1$. Show that $(3, -1)$ lies on the curve, and decide whether $(3, 5)$ lies on the curve. [5 marks]

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C5 A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. The curve crosses itself at one point. Find the two values of t there and the coordinates of the point. [5 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 A curve C has parametric equations $x = \ln(t + 1)$, $y = t^2 - 4$, $t > -1$.

(a) Show that a Cartesian equation of C is $y = e^{2x} - 2e^x - 3$. [3 marks]

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(b) Find the exact coordinates of the points where C meets the coordinate axes. [4 marks]

Answer .....

(c) Find the equation of the asymptote to C. [2 marks]

Answer .....

Total for D1: 9 marks

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D2 A curve C has parametric equations $x = 3t^2$, $y = 6t$.

(a) Find a Cartesian equation of C. [2 marks]

Answer .....

(b) Show that the line $y = x + 3$ meets C at exactly one point, and find its coordinates. [3 marks]

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(c) The line $y = 2x - 12$ meets C at A and B. Find the coordinates of A and B. [3 marks]

Answer .....

(d) Find the exact length AB. [2 marks]

Answer .....

Total for D2: 10 marks

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D3

> Mia says: "$\sin t = \frac{1}{2}$ gives $t = \frac{\pi}{6}$, so C meets the line $y = \frac{1}{2}$ at one point only."

Mia is wrong.

(a) Find the mistake, and find the coordinates of every point where C meets the line $y = \frac{1}{2}$. [3 marks]

(b) Find a Cartesian equation of C, and describe C. [2 marks]

Answer .....

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(c) The line $x + y = 3$ meets C at two points. Find the values of t at these points and the coordinates of the points. [4 marks]

Answer .....

Total for D3: 9 marks

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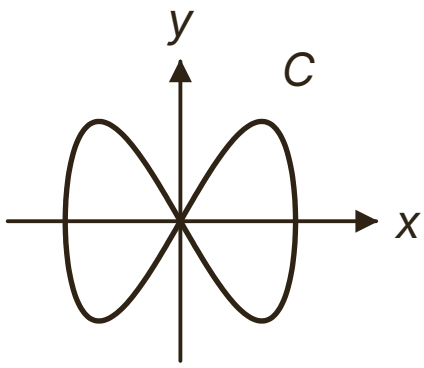
D4 A circle has parametric equations $x = 3 + 5\cos t$, $y = -2 + 5\sin t$, $0 \leq t < 2\pi$. The line $y = 2x + k$, where k is a constant, is a tangent to the circle. Find the possible values of k, giving your answers in the form $a + b\sqrt{5}$. [7 marks]

Answer .....

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D5 The figure shows the curve C with parametric equations $x = \cos t$, $y = \sin 2t$, $0 \leq t < 2\pi$.

(a) Show that C passes through the origin twice, and give the two values of t there. [2 marks]



(b) Find the coordinates of every point where C meets the line $y = x$. [4 marks]

Answer .....

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(c) Find the exact coordinates of the points where C meets the line $y = 1$. [3 marks]

Answer .....

Total for D5: 9 marks

D6 A curve C has parametric equations $x = t^3$, $y = t^2$.

(a) Find the coordinates of the points where C meets the line $y = 2x$. [3 marks]

Answer .....

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(b) The line $y = kx$, where $k > 0$, meets C at the origin and at one other point P. Find the coordinates of P in terms of k. [2 marks]

Answer .....

(c) P lies on the line $x + y = 2$. Find the value of k. [4 marks]

Answer .....

Total for D6: 9 marks

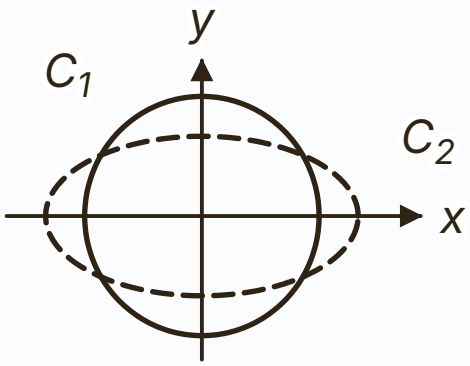
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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The figure shows the curve C_1 with parametric equations $x = 3\cos t$, $y = 3\sin t$, and the curve C_2 with parametric equations $x = 4\cos s$, $y = 2\sin s$, where $0 \leq t < 2\pi$ and $0 \leq s < 2\pi$. The two curves meet at four points, which are the corners of a rectangle. Find the exact area of the rectangle, giving your answer in the form $\dfrac{a\sqrt{35}}{b}$. [8 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write a Cartesian equation for each curve.

Hint 2 Both equations hold at a corner. Subtract to remove x^2 and find y^2, then x^2.

Hint 3 By symmetry the corners are $(\pm x, \pm y)$, so the sides are $2x$ and $2y$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch a parametric curve, marking its end points and where it meets the axes.	○ ○ ○
I can find where a parametric curve meets the axes by setting x = 0 or y = 0.	○ ○ ○
I can find where a parametric curve meets a line by substituting x and y into its equation.	○ ○ ○
I can check every value of the parameter against the interval given.	○ ○ ○

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