Small angle approximations

111 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-small-angle-approximations/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Expand $\left(1 - \frac{\theta^2}{2}\right)^2$, ignoring any terms in θ^3 or higher powers.

Answer .....

A2 Needed today [2 marks]

Write 3° in radians, as a multiple of π.

Answer .....

A3 Needed today [2 marks]

Find the first three terms, in ascending powers of θ, of the binomial expansion of $(1 + 2\theta)^{-1}$.

Answer .....

A4 Common slip [2 marks]

Work out $\sin 0.1$ to 5 decimal places. Make sure your calculator is in radian mode.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Use $\sin\theta \approx \theta$ to estimate $\sin 0.2$. Find the percentage error in this estimate, to 3 significant figures.

Hint The percentage error is (estimate − true value) ÷ true value × 100. Use the true value from your calculator, in radian mode.

B1 Use $\sin\theta \approx \theta$ to estimate $\sin 0.15$. Find the percentage error in this estimate, to 3 significant figures. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

Given that θ is small, show that $\dfrac{1-\cos\theta}{\theta\sin\theta} \approx \dfrac{1}{2}$.

Hint Replace $\cos\theta$ by $1-\dfrac{\theta^2}{2}$ and $\sin\theta$ by θ, then simplify.

B2 Given that θ is small, show that $\dfrac{1-\cos 2\theta}{\theta\tan\theta} \approx 2$. [4 marks]

EXAMPLE 3

Given that θ is small, find an approximation for $3\cos 2\theta + \sin\theta$ in the form $a + b\theta + c\theta^2$.

Hint Replace 2θ as a whole: $\cos 2\theta \approx 1-\dfrac{(2\theta)^2}{2} = 1-2\theta^2$.

B3 Given that θ is small, find an approximation for $4\cos 3\theta - 2\tan\theta$ in the form $a + b\theta + c\theta^2$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

Given that θ is small, use the small angle approximations to find an approximate solution of $2\sin\theta + \cos\theta = 1.1$, to 3 significant figures.

Hint The approximations turn the equation into a quadratic in θ. It has two roots: keep the one that is small.

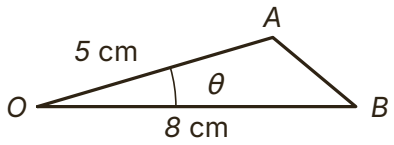
B4 Given that θ is small, use the small angle approximations to find an approximate solution of $3\tan\theta + 2\cos\theta = 2.1$, to 3 significant figures. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 5

In triangle OAB, $OA = 5$ cm, $OB = 8$ cm and angle $AOB = \theta$ radians, where θ is small. Show that $AB^2 \approx 9 + 40\theta^2$.



Hint Use the cosine rule, then replace $\cos\theta$ by $1 - \frac{\theta^2}{2}$.

B5 In triangle OPQ, $OP = 3$ cm, $OQ = 7$ cm and angle $POQ = \theta$ radians, where θ is small. Find an approximation for PQ^2 in the form $a + b\theta^2$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 Amy says that $\sin 4^\circ \approx 4$, because $\sin\theta \approx \theta$ for small angles. Explain her mistake. Then find the percentage error in using $\sin\theta \approx \theta$ for an angle of 4°, to 3 significant figures. [4 marks]

Answer .....

C2 Ben writes: "$\cos 3\theta \approx 1 - \dfrac{3\theta^2}{2}$ when θ is small." Explain his mistake, give the correct approximation, and use it to estimate $\cos 0.3$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C3 Use $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$ to estimate $\cos 1$. Find the percentage error in this estimate, to 3 significant figures, and explain why the error is large. [5 marks]

Answer .....

C4 Given that θ is small, show that $\dfrac{\sin\theta\tan\theta}{1-\cos\theta} \approx 2$. [4 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C5 Use a small angle approximation to estimate $\cos 5°$, to 6 decimal places. Compare it with the value on your calculator. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 This question is about the expression $\cos 4\theta + 2\sin 3\theta$, where θ is small and in radians.

(a) Show that $\cos 4\theta + 2\sin 3\theta \approx 1 + 6\theta - 8\theta^2$. [3 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Use the approximation with $\theta = 0.1$ to estimate $\cos 0.4 + 2\sin 0.3$. [2 marks]

Answer .....

(c) Find the percentage error in your estimate, to 3 significant figures. [2 marks]

Answer .....

(d) Use the approximation to find, to 3 significant figures, the small value of θ for which $\cos 4\theta + 2\sin 3\theta = 1.2$. [2 marks]

Answer .....

Total for D1: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D2

> Kim writes: "When θ is small, $\cos 2\theta \approx 1 - 2\theta$, so $\dfrac{1-\cos 2\theta}{\sin\theta} \approx \dfrac{2\theta}{\theta} = 2$."

Kim is wrong.

(a) Explain the mistake Kim has made. [2 marks]

(b) Find a correct approximation for $\dfrac{1-\cos 2\theta}{\sin\theta}$ in terms of θ. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Hence estimate $\dfrac{1-\cos 0.1}{\sin 0.05}$. [2 marks]

Answer .....

(d) Find the percentage error in your estimate in part (c), to 3 significant figures. [2 marks]

Answer .....

Total for D2: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

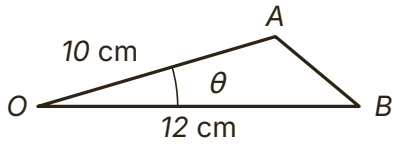
D3 Given that θ is small, find the value that $\dfrac{\cos 2\theta - \cos 3\theta}{\theta \tan \theta}$ is close to. [6 marks]

Show your working.

Answer .....

D4 In triangle OAB, $OA = 10$ cm, $OB = 12$ cm and angle $AOB = \theta$ radians, where θ is small.

(a) Show that $AB^2 \approx 4 + 120\theta^2$. [3 marks]



© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Use the approximation to estimate AB when $\theta = 0.1$, to 4 significant figures. [2 marks]

Answer .....

(c) Find the percentage error in your estimate of AB, to 2 significant figures. [2 marks]

Answer .....

(d) Use the approximation to find the value of θ for which $AB = 3$ cm, to 3 significant figures. [2 marks]

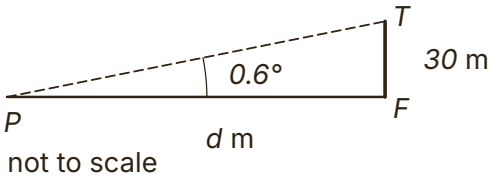
Answer .....

Total for D4: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D5 A tower TF is 30 m tall and stands on level ground. From a point P on the ground, the angle of elevation of the top of the tower T is $0.6°$. The distance from P to the foot of the tower is d m.

(a) Write $0.6°$ in radians, to 3 significant figures. [2 marks]



not to scale

Answer .....

(b) Use $\tan\theta \approx \theta$ to estimate d, to the nearest metre. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Show that the percentage error in this estimate is less than 0.01%. [3 marks]

(d) Explain why the same method would give a much worse estimate if the angle of elevation were $25°$. [2 marks]

Total for D5: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D6 This question is about the expression $\dfrac{2-\cos\theta}{1+\sin\theta}$, where θ is small and in radians.

(a) Show that $\dfrac{2-\cos\theta}{1+\sin\theta} \approx 1-\theta+\dfrac{3}{2}\theta^2$. [4 marks]

(b) Use the approximation to estimate the value of the expression when $\theta = 0.1$. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the percentage error in your estimate, to 3 significant figures. [3 marks]

Answer .....

Total for D6: 9 marks

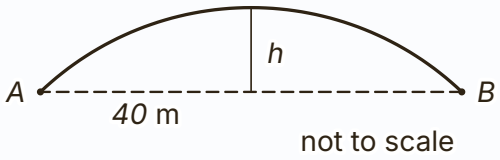
© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A footbridge is an arc of a circle of radius 400 m. Its two ends, A and B, are level and 40 m apart. The height h m is the height of the middle of the bridge above the straight line AB. Use a small angle approximation to show that $h \approx \dfrac{c^2}{8r}$, where c is the length AB and r is the radius, and hence estimate h. [9 marks]



not to scale

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Let the angle AOB at the centre be 2α. Then $h = r - r\cos\alpha$, since the middle of AB is $r\cos\alpha$ from the centre.

Hint 2 α is small, so $\cos\alpha \approx 1 - \dfrac{\alpha^2}{2}$, which gives $h \approx \dfrac{r\alpha^2}{2}$.

Hint 3 Half the chord is $r\sin\alpha$, so $\dfrac{c}{2} \approx r\alpha$, which gives $\alpha \approx \dfrac{c}{2r}$. Put this into h.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use sin θ ≈ θ, tan θ ≈ θ and cos θ ≈ 1 – θ²/2 when θ is small and measured in radians.	○ ○ ○
I can replace kθ by the whole of kθ in an approximation, such as cos 3θ ≈ 1 – 9θ²/2.	○ ○ ○
I can find an approximate expression and its percentage error.	○ ○ ○
I can use the approximations to solve an equation or to simplify the cosine rule.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.