Solving differential equations

112 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-solving-differential-equations/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Find $\int \frac{1}{2y+1}\,dy$.

Answer .....

A2 Common slip [2 marks]

$\ln|y| = 3x + c$. Write y in terms of x using a single constant A.

Answer .....

A3 From chapter 1 [2 marks]

Express $\frac{3}{(x-1)(x+2)}$ in partial fractions.

Answer .....

A4 Needed today [2 marks]

Find $\int xe^x\,dx$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Given that $y = 2$ when $x = 0$, solve $\frac{dy}{dx} = 6x^2y$, $y > 0$, giving y in terms of x.

Hint Move every y to the left with dy and every x to the right with dx. Put one constant on the right only.

B1 Given that $y = 3$ when $x = 0$, solve $\frac{dy}{dx} = y \sin x$, $y > 0$, giving y in terms of x. [4 marks]

Answer .....

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EXAMPLE 2

Given that $y = 2$ when $x = 0$, solve $\dfrac{dy}{dx} = \dfrac{2x+1}{3y^2}$, giving y in terms of x.

Hint Separate to $\int 3y^2\,dy = \int (2x+1)\,dx$. Find the constant first, then make y the subject.

B2 Given that $y = 2$ when $x = 0$, solve $\dfrac{dy}{dx} = \dfrac{xe^x}{y}$, $y > 0$. Give y^2 in terms of x. [5 marks]

Answer .....

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EXAMPLE 3

Given that $y = 4$ when $x = 1$, solve $\frac{dy}{dx} = \frac{2y}{(x+1)(x+3)}$, $x > -1$, $y > 0$, giving y in terms of x.

Hint Write $\frac{2}{(x+1)(x+3)} = \frac{1}{x+1} - \frac{1}{x+3}$ before integrating. Then combine the logs.

B3 Given that $y = 1$ when $x = 2$, solve $\frac{dy}{dx} = \frac{3y}{(x-1)(x+2)}$, $x > 1$, $y > 0$, giving y in terms of x. [5 marks]

Answer .....

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EXAMPLE 4

Given that $y = \frac{\pi}{4}$ when $x = 0$, solve $\frac{dy}{dx} = e^x \cos^2 y$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Hint $\frac{1}{\cos^2 y} = \sec^2 y$, and $\int \sec^2 y \, dy = \tan y$.

B4 [5 marks]

Given that $y = \frac{\pi}{6}$ when $x = \frac{\pi}{3}$, solve $\frac{dy}{dx} = \frac{\sin x}{\cos y}$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$, giving $\sin y$ in terms of x. Hence find y when $x = \frac{\pi}{4}$, to 3 significant figures.

Answer .....

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C1 [5 marks]

Sam solves $\frac{dy}{dx} = 2xy$, $y > 0$, given that $y = 5$ when $x = 0$. He writes: "$\ln y = x^2$, so $y = e^{x^2} + c$, and then $c = 4$." Explain his mistake. Find the correct particular solution, and the value of y at $x = 1$ from his answer and from the correct one, to 3 significant figures.

Answer .....

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C2 Find the general solution of $\frac{dy}{dx} = -\frac{x}{y}$, $y \neq 0$, and describe the family of solution curves. Find the equation of the curve that passes through $(5, -12)$ and state its radius. [5 marks]

Answer .....

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C3 Given that $y = \frac{1}{2}$ when $x = 0$, solve $\frac{dy}{dx} = y^2 e^{-x}$, giving y in terms of x. State what happens to y as x becomes large. [6 marks]

Answer .....

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C4 Ben solves $\frac{dy}{dx} = 3(y-2)$, given that $y = 1$ when $x = 0$. He writes "$\ln(y-2) = 3x + c$" and then says: "At $x = 0$ this needs $\ln(-1)$, which does not exist, so there is no solution." Explain his mistake and find the particular solution. [5 marks]

Answer .....

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D1 $f(x) = \dfrac{4x+5}{(x+2)(2x+1)}$, $x \geq 0$.

(a) Express $f(x)$ in partial fractions. [3 marks]

Answer .....

(b) Given that $y = 6$ when $x = 0$, solve $\dfrac{dy}{dx} = yf(x)$, $y > 0$. Give y as a polynomial in x. [5 marks]

Answer .....

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(c) Find the value of x for which $y = 27$. [2 marks]

Answer .....

Total for D1: 10 marks

D2

Priya solves $\frac{dy}{dx} = \frac{xy}{x^2 + 4}$, $y > 0$, given that $y = 2$ when $x = 0$. She writes: "$\ln y = \ln(x^2 + 4) + c$, so $y = x^2 + 4 + c$. At $x = 0$, $2 = 4 + c$, so $y = x^2 + 2$."

Priya makes two mistakes.

(a) Explain her first mistake. [2 marks]

(b) Explain her second mistake. [2 marks]

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(c) Find the correct particular solution. [4 marks]

Answer .....

(d) Show that the correct solution curve passes through $(\sqrt{5}, 3)$, and that Priya's curve does not. [2 marks]

Total for D2: 10 marks

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D3 Given that $y = 1$ when $x = 0$, solve $\dfrac{dy}{dx} = e^{2x-y}$, giving y in terms of x. Hence find the exact value of x for which $y = 2$. [8 marks]

Answer .....

D4 A curve satisfies $\dfrac{dy}{dx} = \dfrac{4\cos^2 x}{y}$, $y > 0$, and passes through $(0, 2)$.

(a) Show that $4\cos^2 x = 2 + 2\cos 2x$. [2 marks]

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(b) Hence find y^2 in terms of x. [5 marks]

Answer .....

(c) Find y when $x = \frac{\pi}{4}$, giving your answer to 3 significant figures. [2 marks]

Answer .....

Total for D4: 9 marks

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D5 This question is about the differential equation $\frac{dy}{dx} = \frac{x}{y\sqrt{x^2+5}}$, $y > 0$.

(a) Use the substitution $u = x^2 + 5$ to find $\int \frac{x}{\sqrt{x^2+5}}\,dx$. [3 marks]

Answer .....

(b) Given that $y = 3$ when $x = 2$, find y^2 in terms of x. [4 marks]

Answer .....

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(c) Find the exact positive value of x for which $y = \sqrt{11}$. [2 marks]

Answer .....

Total for D5: 9 marks

D6 This question is about the differential equation $\frac{dy}{dx} = y \tan x$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) Find the general solution, in the form $y = f(x)$. [4 marks]

Answer .....

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(b) Find the particular solution through $(0, 3)$ and the exact value of y on it when $x = \frac{\pi}{3}$. [2 marks]

Answer .....

(c) Explain why this solution cannot be extended to $x = \frac{\pi}{2}$. [2 marks]

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [10 marks]

The curve C satisfies $\dfrac{dy}{dx} = e^{x+y}$ and passes through the origin. Find y in terms of x. Show that C has a vertical asymptote and find its equation. Find the exact x-coordinate of the point on C where the gradient is 4.

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $e^{x+y} = e^x e^y$, so the variables separate: $\int e^{-y}\,dy = \int e^x\,dx$.

Hint 2 You should reach $e^{-y} = 2 - e^x$. What happens to y as e^x gets close to 2?

Hint 3 The gradient is $e^x e^y$, and $e^y = \dfrac{1}{2 - e^x}$. Set the gradient equal to 4 and solve for e^x.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can separate the variables and integrate both sides, putting one constant on one side.	○ ○ ○
I can take exponentials correctly, so the constant added to ln|y| becomes a constant A that multiplies, not one that is added.	○ ○ ○
I can use a given point to find a particular solution, and leave it in a clear form.	○ ○ ○
I can use partial fractions, integration by parts and trigonometric identities to integrate each side.	○ ○ ○

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