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mathswariz.uk/worksheets/a-level-solving-equations-with-logarithms-and-natural-logarithms/

Solving equations with logarithms, and natural logarithms

70 marks

Date _____

A Do Now

Quick questions to start.

- A1** Last lesson [2 marks] Solve $\log_3(2x - 1) = 2$.
- A2** Needed today [2 marks] Write $2^6 = 64$ as a logarithm. Find $\log_3 243$.

Answer

Answer

- A3** GCSE Higher [2 marks] Solve $u^2 - 5u - 14 = 0$.
- A4** This chapter [2 marks] $P = 250e^{0.04t}$. Find P when $t = 10$, to the nearest whole number.

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve, giving each answer to 3 significant figures. (a) $4^x = 30$ (b) $3^{2x+1} = 50$

Hint $a^x = b$ means $x = \log_a b$. In (b), first find what the power $2x + 1$ must be.

B1 Solve, giving each answer to 3 significant figures. (a) $6^x = 45$ (b) $5^{3x-2} = 80$ [3 marks]

Answer

EXAMPLE 2

Solve $2^{2x} - 9(2^x) + 14 = 0$, giving answers to 3 significant figures where needed.

Hint $2^{2x} = (2^x)^2$, so this is a quadratic in 2^x . Let $y = 2^x$.

B2 Solve $3^{2x} - 13(3^x) + 30 = 0$, giving answers to 3 significant figures where needed. [3 marks]

Answer

EXAMPLE 3

Solve $5^x = 3^{x+2}$, giving your answer to 3 significant figures.

Hint Take $\ln$ of both sides, then use the power law. Collect the terms in x on one side and factorise.

B3 Solve $7^x = 2^{x+3}$, giving your answer to 3 significant figures. [3 marks]

Answer

EXAMPLE 4

Solve, giving exact answers. (a)

$$e^{3x-1} = 8 \quad \text{(b)} \quad 3 \ln(2x) + 1 = 7$$

Hint $\ln$ undoes e , and e undoes $\ln$. Get the e or the $\ln$ on its own first.

B4 Solve, giving exact answers. (a) $e^{2x+5} = 9$ [3 marks]

$$\text{(b)} \quad \ln(5x - 2) = 3$$

Answer

c Practice

A-Level

C1 Solve $e^{2x} - 3e^x - 10 = 0$, giving your answer in exact form. [3 marks]

Answer

C2 Solve $2e^x - 5 = 3e^{-x}$, giving your answer in exact form. [4 marks]

Answer

C3 Solve $(\ln x)^2 + \ln x - 6 = 0$, giving your answers in exact form. [3 marks]

Answer

C4 On one set of axes, sketch $y = e^x$ and $y = \ln x$. Show the line each graph is a reflection of the other in, the point where each graph crosses an axis, and the asymptote of each graph. [4 marks]

D Exam-style questions

Edexcel exam style

D1 The mass, m grams, of a radioactive substance t days after it is made is modelled by $m = 80e^{-0.03t}$. A second substance has mass $m = 50e^{-0.01t}$ grams t days after it is made. Both are made at the same time.

(a) Find, to the nearest day, the time it takes the first substance to fall to 20 grams. [3 marks]

Answer

- (b) Show that the two substances have the same mass when $t = 50 \ln 1.6$, and find this time to 1 decimal place. [4 marks]

- (c) Hence state the times for which the first substance has the greater mass. [1 mark]

Answer

Total for D1: 8 marks

D2

Leo says: " $\ln(x^2) = 2 \ln x$, so $\ln(x^2) = 4$ means $\ln x = 2$, and the only solution is $x = e^2$."

Leo is wrong.

- (a) Show that $x = -e^2$ also satisfies $\ln(x^2) = 4$. [2 marks]

- (b) Explain why Leo's step loses a solution. Then solve
 $\ln(x^2) = 2 \ln(x + 3)$.

[4 marks]

Answer

Total for D2: 6 marks

D3 An equation is $4^{x+1} - 17(2^x) + 4 = 0$.

- (a) Show that, with $y = 2^x$, the equation can be written as
 $4y^2 - 17y + 4 = 0$.

[3 marks]

- (b) Hence solve the equation, without a calculator.

[3 marks]

Answer

- (c) Hence solve $4^{x+1} - 17(2^x) + 4 < 0$.

[2 marks]

Answer

Total for D3: 8 marks

D4 This question is about the equation $\ln(2x + 3) + \ln(x - 1) = 2 \ln(x + 1)$.

- (a) Solve the equation, giving your answer in the form $\frac{a + \sqrt{b}}{2}$. [5 marks]
Explain why it has only one solution.

Answer

- (b) Hence solve $\ln(2e^y + 3) + \ln(e^y - 1) = 2 \ln(e^y + 1)$. Give your answer exactly, and to 3 significant figures. [3 marks]

Answer

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find, in exact form, the two values of x for which $x^{\ln x} = e^4 x^3$. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Take $\ln$ of both sides. The power law turns $\ln(x^{\ln x})$ into $\ln x \times \ln x$.
- Hint 2** The right side becomes $4 + 3 \ln x$, so $(\ln x)^2 = 4 + 3 \ln x$. Let $u = \ln x$.
- Hint 3** $u^2 - 3u - 4 = 0$, so $(u - 4)(u + 1) = 0$. Then $x = e^4$ or $x = e^{-1}$. Both are positive, so both work.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve an equation with the unknown in a power by taking logarithms. ○ ○ ○

I can solve a quadratic in a power such as 2 to the x or e to the x , and reject a power that would be negative. ○ ○ ○

I can solve equations with e and $\ln$ and give exact answers. ○ ○ ○

I can use an exponential model to find when a quantity reaches a value. ○ ○ ○