Solving geometric problems with vectors

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-solving-geometric-problems-with-vectors/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find the distance between the points $(1, 2, -2)$ and $(3, -4, 1)$.

Answer .....

A2 Needed today [2 marks]

$\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\overrightarrow{OB} = 4\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$. Find the position vector of the midpoint of AB.

Answer .....

A3 Common slip [2 marks]

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P lies on AB with $AP : PB = 1 : 2$. Write $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer .....

A4 From Year 1 [2 marks]

$\mathbf{a}$ and $\mathbf{b}$ are not parallel. $(p + 2)\mathbf{a} + 3\mathbf{b} = 5\mathbf{a} + (q - 1)\mathbf{b}$. Find p and q.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Show that the points $A(1, 2, 3)$, $B(3, 5, 4)$ and $C(7, 11, 6)$ are collinear.

Hint Find two vectors that share a point, such as $\overrightarrow{AB}$ and $\overrightarrow{AC}$. If one is a multiple of the other, the lines are parallel, and the shared point puts all three on one line.

B1 Show that $P(2, -1, 4)$, $Q(5, 1, 2)$ and $R(11, 5, -2)$ are collinear, and find the ratio $PQ : QR$. [4 marks]

Answer .....

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EXAMPLE 2

$ABCD$ is a parallelogram. A, B and C are the points $(1, 0, 2)$, $(4, 1, 3)$ and $(6, 4, 1)$. Find the coordinates of D.

Hint Going round $ABCD$ in order, $\overrightarrow{AD} = \overrightarrow{BC}$. Take care with the order of the letters.

B2 $ABCD$ is a parallelogram with $A(2, -1, 1)$, $B(5, 3, 0)$ and $C(4, 6, -2)$. Find the coordinates of D and the exact length of the diagonal BD. [4 marks]

Answer .....

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EXAMPLE 3

A is $(1, 4, -2)$ and B is $(7, 1, 4)$. The point P lies on AB with $AP : PB = 1 : 2$. Find the coordinates of P.

Hint $AP : PB = 1 : 2$ means P is $\frac{1}{3}$ of the way from A to B, so $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$.

B3 [4 marks]

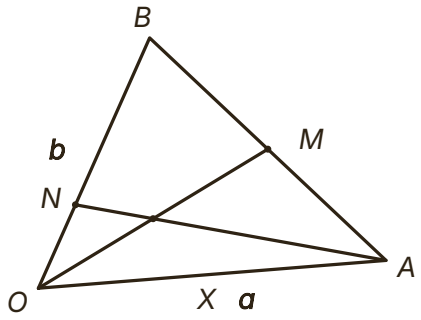
A is $(2, 5, -1)$ and B is $(12, -5, 9)$. The point P lies on AB with $AP : PB = 3 : 2$. Find the coordinates of P and the exact distance OP.

Answer .....

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EXAMPLE 4

In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of AB and N lies on OB with $ON : NB = 1 : 2$. The lines OM and AN meet at X. Find $\overrightarrow{OX}$.



Hint Write $\overrightarrow{OX}$ in two ways: as $\lambda\overrightarrow{OM}$, and as $\overrightarrow{OA} + \mu\overrightarrow{AN}$. As $\mathbf{a}$ and $\mathbf{b}$ are not parallel, the coefficients of $\mathbf{a}$ must match and so must the coefficients of $\mathbf{b}$.

B4 In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P lies on OA with $OP : PA = 2 : 1$, and Q is the midpoint of AB. The lines OQ and BP meet at R. Find $\overrightarrow{OR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, and the ratio $OR : RQ$. [6 marks]

Answer .....

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EXAMPLE 5

$\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are not coplanar. $(2p+q)\mathbf{a}+(q-r)\mathbf{b}+(p+r)\mathbf{c}=7\mathbf{a}+\mathbf{b}+4\mathbf{c}$. Find p, q and r.

Hint Three vectors that are not in one plane cannot be combined to give zero unless every coefficient is zero. So the coefficients of $\mathbf{a}$, of $\mathbf{b}$ and of $\mathbf{c}$ match separately.

B5 $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are not coplanar. $(p+q)\mathbf{a}+(2q-r)\mathbf{b}+(p+2r)\mathbf{c}=5\mathbf{a}+4\mathbf{b}+9\mathbf{c}$. Find p, q and r. [4 marks]

Answer .....

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C1 A, B, C and D are the points $(1, 0, 2)$, $(3, 1, 5)$, $(4, 4, 4)$ and $(8, 6, 10)$. Amir writes: "$\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{CD} = 4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k} = 2\overrightarrow{AB}$, so A, B, C and D lie on one straight line." Find his mistake and show that A, B and C are not collinear. [4 marks]

Answer .....

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C2 A is $(4, -2, 7)$ and B is $(12, 6, -1)$. P lies on AB with $AP : PB = 1 : 3$. Beth writes $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$. Explain her mistake and find the coordinates of P. [4 marks]

Answer .....

C3 A is $(2, 1, -3)$, B is $(5, 3, 1)$ and C is $(k, 7, m)$. Given that A, B and C are collinear, find k and m. [4 marks]

Answer .....

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C4 $A(1, 2, 1)$, $B(3, -1, 4)$ and $C(6, 0, 2)$ are three corners of the parallelogram $ABCD$. Sam finds D from $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{OC} - \overrightarrow{OA}$. Explain his mistake, find the coordinates of D, and check your answer using the midpoints of the diagonals. [5 marks]

Answer .....

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C5 O is the origin and A, B and C are the points $(2, -1, 3)$, $(5, 2, 6)$ and $(1, 4, 0)$. Show that $OABC$ is a trapezium, and find the ratio $OA : CB$. [5 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 The points A and B have coordinates $(1, 3, -2)$ and $(5, -1, 6)$.

(a) Find the coordinates of the point P on AB with $AP : PB = 1 : 3$. [2 marks]

Answer .....

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(b) The point C is $(8, -4, 12)$. Show that A, B and C are collinear, and find the ratio $AB : BC$. [4 marks]

Answer .....

(c) Find the exact distance PC. [2 marks]

Answer .....

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(d) Find the coordinates of the point E such that B is the midpoint of AE. [2 marks]

Answer .....

Total for D1: 10 marks

D2

P, Q, R and S are the points $(1, -2, 3)$, $(4, 2, 1)$, $(10, 10, -3)$ and $(7, 6, -1)$. Mia says that $PQRS$ is a parallelogram, because PQ is parallel to RS and QR is parallel to SP.

Mia is wrong.

(a) Find $\overrightarrow{PQ}$ and $\overrightarrow{RS}$, and show that they are parallel. [2 marks]

Answer .....

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(b) Show that P, Q and R lie on one straight line. [3 marks]

(c) Explain why $PQRS$ is not a parallelogram. [2 marks]

(d) Show that S lies on PR and find the ratio $PS : SR$. [2 marks]

Answer .....

Total for D2: 9 marks

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D3 In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of OA, and P lies on AB with $AP : PB = 2 : 1$. The lines OP and BM meet at X.

(a) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2 marks]

Answer .....

(b) Find $\overrightarrow{BM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2 marks]

Answer .....

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(c) Using your answers to (a) and (b), find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [4 marks]

Answer .....

(d) Hence find the ratios $OX : XP$ and $BX : XM$. [2 marks]

Answer .....

Total for D3: 10 marks

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D4 A, B and C are the points $(2, 1, -1)$, $(6, 3, 5)$ and $(5, -2, 4)$. $ABCD$ is a parallelogram whose diagonals meet at M. Find the coordinates of D and of M, find the exact length of the shorter diagonal, and show that $ABCD$ is not a rhombus. [8 marks]

Answer .....

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D5 O is the origin and A, B and C are the points $(4, 2, -2)$, $(3, 6, 2)$ and $(1, 5, 3)$.

(a) Show that $OABC$ is a trapezium. [3 marks]

(b) The diagonals OB and AC meet at X. Find the coordinates of X. [4 marks]

Answer .....

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(c) Hence find the ratio $OX : XB$. [2 marks]

Answer .....

Total for D5: 9 marks

D6 $OABCDEFG$ is a cuboid. $OABC$ is the base, and D, E, F and G are directly above O, A, B and C. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$. Prove, using vectors, that the diagonals OF and AG meet, and that they meet at the midpoint of each. [7 marks]

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [9 marks]

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AB, and N lies on BC with $BN : NC = 1 : 2$. The lines OM and AN meet at X. Find the ratios $OX : XM$ and $AX : XN$.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$. Write $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

Hint 2 $\overrightarrow{OX} = \lambda\overrightarrow{OM}$ and also $\overrightarrow{OX} = \overrightarrow{OA} + \mu\overrightarrow{AN}$.

Hint 3 $\mathbf{a}$ and $\mathbf{c}$ are not parallel, so compare the coefficients of each. λ and μ give the two ratios.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can show that three points are collinear and find the ratio in which one divides the other two.	○ ○ ○
I can find the fourth vertex of a parallelogram and show that a shape is a trapezium.	○ ○ ○
I can find the position vector of a point that divides a line in a given ratio.	○ ○ ○
I can find where two lines meet by comparing coefficients of vectors that are not parallel.	○ ○ ○

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