Solving modulus problems

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-solving-modulus-problems/

Date

A Do Now

Quick questions to start.

A1 Lesson 1 [2 marks]

Solve $|3x + 2| = 7$.

Answer .....

A2 Lesson 4 [2 marks]

Write down the coordinates of the vertex of $y = 4 - 3|x - 2|$, and the point where it meets the y-axis.

Answer .....

A3 From Year 1 [2 marks]

Solve $x^2 - x - 12 \geq 0$.

Answer .....

A4 Needed today [2 marks]

Solve $|x - 1| < 4$.

Answer .....

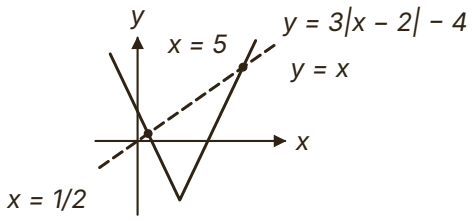
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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve $3|x-2|-4=x$.



Hint Split at $x=2$, where the inside of the modulus is zero. Solve each case, then check each answer lies in its own case.

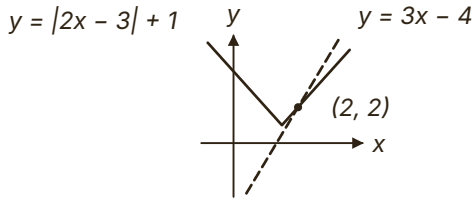
B1 Solve $2|x+3|-5=3-x$. [4 marks]

Answer .....

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EXAMPLE 2

Solve $|2x - 3| + 1 = 3x - 4$.



Hint Write it as $|2x - 3| = 3x - 5$. The right-hand side must be ≥ 0, so any answer with $3x - 5 < 0$ is rejected.

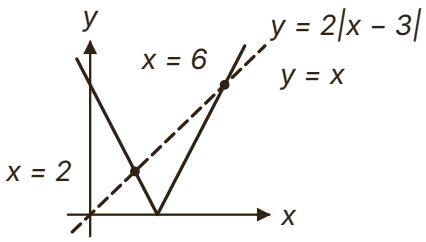
B2 Solve $3|x + 2| - 1 = 5x + 3$. [4 marks]

Answer .....

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EXAMPLE 3

Solve $2|x-3| > x$.



Hint Find where $2|x-3| = x$, then use a sketch to see where the V is above the line.

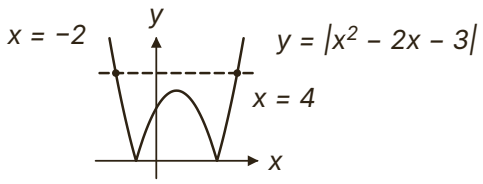
B3 Solve $3|x+1| - 2 \leq x + 4$. [4 marks]

Answer .....

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EXAMPLE 4

Solve $|x^2 - 2x - 3| = 5$.



Hint Solve $x^2 - 2x - 3 = 5$ and $x^2 - 2x - 3 = -5$. A case with no real roots gives no solutions.

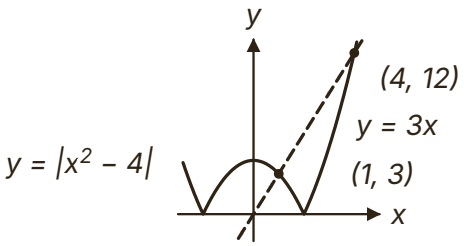
B4 Solve $|x^2 - 4x| = 4$. Give your answers in exact form. [4 marks]

Answer .....

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EXAMPLE 5

Solve $|x^2 - 4| < 3x$.



Hint $|A| < B$ means $-B < A < B$. Solve each half, then take the values that satisfy both. Since $|x^2 - 4| \geq 0$, any solution must have $3x > 0$.

B5 Solve $|x^2 - 9| < 4x$. Give your answer in exact form. [5 marks]

Answer .....

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C1 Mia solves $4|x-1|-3=2x-7$. She writes: "$4(x-1)=2x-4$ gives $x=0$, and $-4(x-1)=2x-4$ gives $x=\frac{4}{3}$. So $x=0$ or $x=\frac{4}{3}$." [5 marks]

(a) Show that neither of Mia's values satisfies the equation.

(b) Explain, by considering the sign of $2x-4$, why the equation has no solutions.

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C2 Ravi solves $|2x - 1| \geq x + 2$. He finds the critical values $-\frac{1}{3}$ and 3 and writes $-\frac{1}{3} \leq x \leq 3$. [4 marks]

(a) Test $x = 0$ in the inequality to show Ravi is wrong.

(b) Write down the correct solution.

Answer .....

C3 Solve $|x^2 - 7x + 6| = 6$. [4 marks]

Answer .....

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C4 Solve $|x^2 - 2x - 4| < 4$. [5 marks]

Answer .....

C5 Solve $|3x - 2| = x^2 - 2$. [5 marks]

Answer .....

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D1 The function f is defined by $f(x) = 3|x - 2| - 6$, $x \in \mathbb{R}$.

(a) Sketch the graph of $y = f(x)$. Give the coordinates of its vertex and of the points where it meets the axes. [3 marks]

(b) Solve $f(x) = x - 2$. [4 marks]

Answer .....

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(c) Hence solve $f(x) < x - 2$. [2 marks]

Answer .....

(d) Find the value of c for which $f(x) = x + c$ has exactly one solution. [2 marks]

Answer .....

Total for D1: 11 marks

D2

> Zara solves $|x^2 - 3x - 10| = 2x$. She solves $x^2 - 3x - 10 = 2x$ and $x^2 - 3x - 10 = -2x$, and gives all four roots as solutions.

Zara is wrong.

(a) Explain why every solution must satisfy $x \geq 0$. [2 marks]

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(b) Find the exact solutions of the equation. [5 marks]

Answer .....

Total for D2: 7 marks

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D3 Find the set of values of x for which $|x^2 - 5x| > 2x - 6$. Give your answer in exact form. [9 marks]

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D4 The functions f and g are defined by $f(x) = 2|x + 1|$ and $g(x) = |x - 3| + 2$, $x \in \mathbb{R}$.

(a) On the same axes, sketch $y = f(x)$ and $y = g(x)$. Give the coordinates of each vertex and of the points where each graph meets the y-axis. [3 marks]

(b) Solve $f(x) = g(x)$. [4 marks]

Answer .....

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(c) Hence solve $f(x) \leq g(x)$. [2 marks]

Answer .....

Total for D4: 9 marks

D5 The function h is defined by $h(x) = |x^2 + 2x - 8|$, $x \in \mathbb{R}$.

(a) Sketch the graph of $y = h(x)$. Give the coordinates of the points where it meets the axes, and of its maximum point. [3 marks]

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(b) Solve $h(x) = 5$. Give your answers in exact form. [4 marks]

Answer .....

(c) Find the values of k for which $h(x) = k$ has exactly two solutions. [2 marks]

Answer .....

Total for D5: 9 marks

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D6 The curve $y = |x^2 - 1|$ and the line $y = x + 1$ are drawn on the same axes.

(a) Find the x-coordinates of the points where they meet. [4 marks]

Answer .....

(b) Hence solve $|x^2 - 1| \leq x + 1$. [3 marks]

Answer .....

Total for D6: 7 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find the set of values of k for which the equation $|x^2 - 4x + 3| = kx$ has exactly four real solutions. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Sketch $y = |x^2 - 4x + 3|$. It meets the x-axis at 1 and 3, and the part between them is reflected so it has a maximum at $(2, 1)$. The line $y = kx$ passes through the origin.

Hint 2 For $k > 0$ the line meets the curve once to the left of $x = 1$ and once to the right of $x = 3$. The other two solutions must come from the reflected part.

Hint 3 The largest k is when the line touches the reflected part: $-(x^2 - 4x + 3) = kx$ has a repeated root, so its discriminant is zero.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve an equation such as a|x − b| + c = mx + d by splitting it into two cases and rejecting answers that do not fit.	○ ○ ○
I can solve an inequality with a modulus by finding the crossing points and reading the solution from a sketch.	○ ○ ○
I can solve |f(x)| = k and |f(x)| < g(x) when f is a quadratic.	○ ○ ○
I can use a sketch to find the values of k that give a set number of solutions.	○ ○ ○

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