

Solving quadratics and completing the square

70 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-solving-quadratics-completing-square/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] **A2** Last lesson [2 marks]
Factorise $x^2 - 3x - 70$. Simplify $\sqrt{108} - \sqrt{12}$.

Answer

Answer

- A3** GCSE Higher [2 marks] **A4** From chapter 1 [2 marks]
Expand and simplify Factorise completely
 $(x - 6)^2$. $2x^3 - 18x$.

Answer

Answer

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve (a) $10x^2 = 9 - 9x$ (b)
 $4x^2 = 7x$.

Hint Make one side 0 first. (a) $ac = -90$: use 15 and -6 . (b) Factorise; do not divide by x .

- B1** Solve (a) $8x^2 = 2x + 15$ [4 marks]
(b) $3x^2 = 11x$.

Answer (a) (b)

EXAMPLE 2

Solve $2x^2 - 7x - 3 = 0$. Give your answers exactly, then correct to 3 significant figures.

Hint $a = 2$, $b = -7$, $c = -3$, so
 $b^2 - 4ac = 49 + 24$.

B2 Solve $3x^2 + 4x - 5 = 0$. [4 marks]

Give exact answers in the form $\frac{p \pm \sqrt{q}}{3}$, then answers to 3 s.f.

Answer ;

EXAMPLE 3

Complete the square for (a)
 $x^2 - 9x + 3$ (b) $3x^2 - 24x + 41$.

Hint Halve the coefficient of x . In (b) take out 3 from the first two terms.

B3 Complete the square [4 marks]

for (a) $x^2 + 5x + 1$
 (b) $2x^2 - 20x + 9$.

Answer (a) (b)

EXAMPLE 4

Solve, giving exact answers, (a)
 $(3x + 1)^2 = 20$ (b) $x^2 - 8x + 4 = 0$
by completing the square.

Hint Square-root with $\pm$. In (b)

$(x - 4)^2 - 16 + 4 = 0$, so $(x - 4)^2 = 12$.

B4 Solve, giving exact answers, (a) [5 marks]

$(2x - 5)^2 = 18$ (b)
 $2x^2 - 10x + 3 = 0$ by
completing the square.

Answer (a) (b)

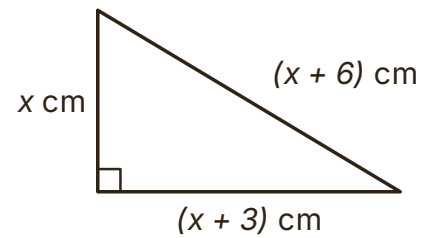
c Practice

A-Level

C1 Solve $\frac{x + 2}{x - 1} = 2x$. [4 marks]

Answer $x = \dots\dots$

- C2** The diagram shows a right-angled triangle. Its sides are x cm, $(x + 3)$ cm and $(x + 6)$ cm. Find x . [4 marks]



Not drawn accurately

Answer $x = \dots\dots$

- C3** The solutions of $x^2 + bx + c = 0$ are $x = 5 \pm \sqrt{2}$. Find b and c . [3 marks]

Answer $b = \dots$ $c = \dots$

D Exam-style questions

Edexcel exam style

- D1** Mia is asked to write $2x^2 + 12x + 7$ in the form $a(x + b)^2 + c$. She writes:
 $2x^2 + 12x + 7 = 2(x + 3)^2 - 9 + 7 = 2(x + 3)^2 - 2$.

Mia has made a mistake.

- (a) Explain the mistake Mia has made. [1 mark]

- (b) Write $2x^2 + 12x + 7$ correctly in the form $a(x + b)^2 + c$. [2 marks]

Answer

- (c) Hence solve $2x^2 + 12x + 7 = 0$, giving your answers in the form $p \pm \frac{\sqrt{q}}{2}$. [2 marks]

Answer

Total for D1: 5 marks

D2 $f(x) = 18 + 12x - 2x^2$.

- (a) Write $f(x)$ in the form $p - q(x - r)^2$, where p , q and r are integers. [3 marks]

Answer

- (b) Hence solve $f(x) = 0$, giving your answers in the form $a \pm b\sqrt{2}$. [2 marks]

Answer

- (c) Hence write down the maximum value of $f(x)$, and the value of x at which it occurs. [2 marks]

Answer maximum at $x = \dots\dots$

(d) Hence solve $f(x) = 4$.

[2 marks]

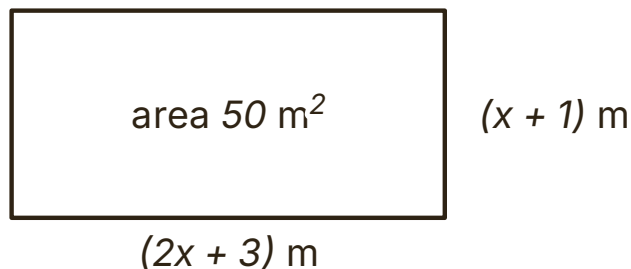
Answer $x = \dots\dots$

Total for D2: 9 marks

D3 A rectangular lawn measures $(2x + 3)$ m by $(x + 1)$ m. Its area is 50 m^2 .

(a) Show that [2 marks]

$$2x^2 + 5x - 47 = 0.$$



Not drawn accurately

(b) Hence find the value of x , correct to 3 significant figures.

[3 marks]

Answer $x = \dots\dots$

(c) Explain why the equation gives only one value of x for this lawn. [1 mark]

- (d) Find the perimeter of the lawn, correct to 3 significant figures. [2 marks]

Answer m

Total for D3: 8 marks

D4 Solve the equation $x - 4 = \sqrt{x + 8}$.

- (a) Show that squaring both sides gives $x^2 - 9x + 8 = 0$. [2 marks]

- (b) Hence solve $x^2 - 9x + 8 = 0$. [2 marks]

Answer $x = \dots\dots$

- (c) Check each value in the original equation, and so solve $x - 4 = \sqrt{x + 8}$. [2 marks]

Answer $x = \dots\dots$

Total for D4: 6 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A rectangle has a perimeter of 30 cm and a diagonal of 12 cm. Find [6 marks]
the exact lengths of its sides.

Answer cm and cm

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Call the sides x and y . Write one equation from the perimeter and one from the diagonal.
- Hint 2** $y = 15 - x$, so $x^2 + (15 - x)^2 = 144$.
- Hint 3** $2x^2 - 30x + 81 = 0$. Use the formula and simplify $\sqrt{252}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve a quadratic by factorising or by square-rooting. ☐ ☐ ☐

I can use the quadratic formula and give exact or rounded answers. ☐ ☐ ☐

I can complete the square and use it to solve an equation. ☐ ☐ ☐

I can form a quadratic from a problem and reject roots that do not fit. ☐ ☐ ☐