Solving trigonometric equations in radians

112 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-solving-trigonometric-equations-in-radians/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Solve $\sin x = \frac{1}{2}$ for $0° \leq x \leq 360°$.

Answer .....

A2 Needed today [2 marks]

Write $225°$ in radians, as a multiple of π.

Answer .....

A3 Needed today [2 marks]

Write $2\cos^2 x + 3\sin x$ in terms of $\sin x$ only.

Answer .....

A4 Common slip [2 marks]

Given that $0 \leq x \leq 2\pi$, find the interval that $2x + \frac{\pi}{3}$ lies in.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve $\sin x = -0.4$ for $0 \leq x \leq 2\pi$, giving your answers to 3 significant figures.

Hint The calculator gives one value, which may be outside the interval. Sine is negative in the third and fourth quarters, so use $\pi + \alpha$ and $2\pi - \alpha$ where $\alpha = \sin^{-1} 0.4$.

B1 Solve $\cos x = -0.3$ for $0 \leq x \leq 2\pi$, giving your answers to 3 significant figures. [4 marks]

Answer .....

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EXAMPLE 2

Solve $\tan\left(x + \frac{\pi}{6}\right) = \sqrt{3}$ for $0 \leq x \leq 2\pi$.

Hint Let $u = x + \frac{\pi}{6}$. Change the interval first: $\frac{\pi}{6} \leq u \leq \frac{13\pi}{6}$. Solve for u, then take $\frac{\pi}{6}$ away.

B2 Solve [4 marks]

$2\cos\left(x - \frac{\pi}{4}\right) = -1$

for $0 \leq x \leq 2\pi$,

giving exact answers.

Answer .....

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EXAMPLE 3

Solve $\sin 2x = \dfrac{\sqrt{3}}{2}$ for $0 \leq x \leq \pi$.

Hint If $0 \leq x \leq \pi$ then $0 \leq 2x \leq 2\pi$. Find every value of $2x$ in that interval before you halve.

B3 [5 marks]

Solve $\cos 3x = \dfrac{\sqrt{2}}{2}$ for $0 \leq x \leq \pi$, giving exact answers.

Answer .....

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EXAMPLE 4

Solve $2\cos^2 x + 3\sin x = 3$ for $0 \leq x \leq 2\pi$.

Hint Replace $\cos^2 x$ by $1 - \sin^2 x$ so that the equation has only $\sin x$ in it. Then it is a quadratic in $\sin x$.

B4 Solve $2\sin^2 x = 3\cos x + 3$ for $0 \leq x \leq 2\pi$, giving exact answers. [5 marks]

Answer .....

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EXAMPLE 5

Solve $3\sin x = 2\tan x$ for $0 \leq x \leq 2\pi$, giving your answers to 3 significant figures where needed.

Hint Write $\tan x = \dfrac{\sin x}{\cos x}$, multiply by $\cos x$ and bring everything to one side. Do not divide by $\sin x$: factorise it out.

B5 Solve $5\sin x = 4\tan x$ for $0 \leq x \leq 2\pi$, giving your answers to 3 significant figures where needed. [5 marks]

Answer .....

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C1 Sam solves $2\sin x\cos x = \sin x$ for $0 \le x \le 2\pi$. He divides both sides by $\sin x$ to get $\cos x = \dfrac{1}{2}$, and gives $x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$. Explain his mistake and give every solution. [5 marks]

Answer .....

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C2 Solve $\cos\left(2x - \frac{\pi}{3}\right) = 0.4$ for $0 \leq x \leq \pi$, giving your answers to 3 significant figures. [5 marks]

Answer .....

C3 Zoe solves $\tan x = 2$ for $0 \leq x \leq 2\pi$ and gives $x = 63.4$ and $x = 243.4$. Explain what she has done wrong and give the correct answers to 3 significant figures. [4 marks]

Answer .....

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C4 Solve $\tan^2 x = 3$ for $-\pi \leq x \leq \pi$, giving exact answers. [4 marks]

Answer .....

C5 Solve $2\sin\left(\frac{x}{2}\right) = 1$ for $0 \leq x \leq 4\pi$, giving exact answers. [4 marks]

Answer .....

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D1 This question is about the equation $4\sin^2 x + 8\cos x - 7 = 0$.

(a) Show that the equation can be written as $4\cos^2 x - 8\cos x + 3 = 0$. [2 marks]

(b) Hence solve $4\sin^2 x + 8\cos x - 7 = 0$ for $0 \leq x \leq 2\pi$, giving exact answers. [3 marks]

Answer .....

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(c) Hence solve $4\sin^2 2\theta + 8\cos 2\theta - 7 = 0$ for $0 \leq \theta \leq 2\pi$, giving exact answers. [4 marks]

Answer .....

Total for D1: 9 marks

D2

Leo solves $\sin 2x = \frac{1}{2}$ for $0 \leq x \leq 2\pi$. He writes: "$2x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so $x = \frac{\pi}{12}$ or $\frac{5\pi}{12}$."

Leo is wrong.

(a) Explain the mistake Leo has made. [2 marks]

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(b) Solve $\sin 2x = \frac{1}{2}$ correctly for $0 \leq x \leq 2\pi$. [3 marks]

Answer .....

(c) Solve $\sin 2x = \frac{1}{2}$ for $-\pi \leq x \leq 0$. [2 marks]

Answer .....

Total for D2: 7 marks

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D3 Solve $5\sin 3\theta = 2\cos 3\theta$ for $0 \leq \theta < \pi$, giving your answers to 3 significant figures. [6 marks]

Answer .....

D4 The depth of water, d metres, in a harbour is modelled by $d = 6 + 2.5\sin\left(\frac{\pi t}{6} - \frac{\pi}{3}\right)$, where t is the time in hours after midnight and $0 \leq t \leq 12$.

(a) Find the greatest depth of water and the time it happens. [2 marks]

Answer .....

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(b) Find the two times when the depth is 7 m. Give each time to the nearest minute. [4 marks]

Answer .....

(c) The harbour master says that between midnight and noon the depth is at least 7 m for more than a third of the time. Is she right? Show your working. [3 marks]

Answer .....

Total for D4: 9 marks

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D5 This question is about the equation $2\cos x = 3\tan x$.

(a) Show that the equation can be written as $2\sin^2 x + 3\sin x - 2 = 0$. [3 marks]

(b) Hence solve $2\cos x = 3\tan x$ for $0 \leq x \leq 2\pi$, giving exact answers. Explain why one factor gives no solutions. [4 marks]

Answer .....

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(c) Hence solve $2\cos 2\theta = 3\tan 2\theta$ for $0 \leq \theta \leq \pi$. [2 marks]

Answer .....

Total for D5: 9 marks

D6 This question is about the equation $2\sin^2 x - \cos x = k$, where k is a constant.

(a) Show that the equation can be written as $2\cos^2 x + \cos x + k - 2 = 0$. [2 marks]

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(b) Given that $k = 1$, solve the equation for $0 \leq x \leq 2\pi$, giving exact answers. [4 marks]

Answer .....

(c) Find the set of values of k for which the equation has at least one solution. [4 marks]

Answer .....

Total for D6: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Solve $\sin 3x = \cos 2x$ for $0 \leq x \leq \pi$, giving exact answers. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $\cos 2x$ as a sine: $\cos 2x = \sin\left(\frac{\pi}{2} - 2x\right)$.

Hint 2 If $\sin A = \sin B$ then either $A = B + 2n\pi$ or $A = \pi - B + 2n\pi$, for a whole number n.

Hint 3 Each case gives a formula for x with n in it. Try $n = 0, 1, 2, \ldots$ and $n = -1, \ldots$ and keep only the values between 0 and π.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find every solution of sin x = k, cos x = k or tan x = k in a given interval, in radians.	○ ○ ○
I can change the interval when the angle is 2x, x/2 or x + a, and change back at the end.	○ ○ ○
I can use sin²x + cos²x = 1 or tan x = sin x / cos x to make a quadratic in one ratio and solve it.	○ ○ ○
I can avoid losing solutions by factorising instead of dividing.	○ ○ ○

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