Solving trigonometric equations

115 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-solving-trigonometric-equations-with-identities/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Expand $\sin(x + 30^\circ)$, giving exact values for $\sin 30^\circ$ and $\cos 30^\circ$.

Answer .....

A2 From chapter 6 [2 marks]

Solve $\sec x = 2$ for $0^\circ \leq x < 360^\circ$.

Answer .....

A3 Common slip [2 marks]

Solve $\tan x = -1$ for $0^\circ \leq x < 360^\circ$.

Answer .....

A4 Today's idea [3 marks]

Solve $\cos 2x = \frac{1}{2}$ for $0^\circ \leq x < 180^\circ$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve $\sin(x + 30^\circ) = 2\cos x$ for $0^\circ \leq x < 360^\circ$.

Hint Expand the left side, put the sin x terms on one side and the cos x terms on the other, then divide by cos x.

B1 Solve $2\sin(x - 60^\circ) = \cos x$ for $0^\circ \leq x < 360^\circ$. Give your answers to 1 decimal place. [5 marks]

Answer .....

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EXAMPLE 2

Solve $3\cos 2x + \cos x + 2 = 0$ for $0° \leq x < 360°$.

Hint Use cos 2x = 2 cos²x – 1, so that only cos x is left.

B2 Solve $5\cos 2x + 3\sin x = 1$ for $0° \leq x < 360°$. Give your answers to 1 decimal place where needed. [5 marks]

Answer .....

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EXAMPLE 3

Solve $2\tan^2 x + \sec x = 1$ for $0° \leq x < 360°$.

Hint Use $\tan^2 x = \sec^2 x - 1$, so that only sec x is left.

B3 Solve $\operatorname{cosec}^2 \theta - 3\cot\theta - 5 = 0$ for $0° \leq \theta < 360°$. Give your answers to 1 decimal place where needed. [5 marks]

Answer .....

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EXAMPLE 4

Solve $\sin(2x + 30^\circ) = \frac{1}{2}$ for $0^\circ \leq x < 360^\circ$.

Hint Find the interval for 2x + 30° first. It runs from 30° to 750°.

B4 Solve $\cos(2x - 40^\circ) = 0.3$ for $0^\circ \leq x < 360^\circ$. Give your answers to 1 decimal place. [5 marks]

Answer .....

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EXAMPLE 5

Solve $4\sin x\cos x = \sqrt{3}$ for $0 \leq x < 2\pi$.

Hint 4 sin x cos x is 2 sin 2x. Then 2x runs from 0 to 4π.

B5 Solve $\cos^2 x - \sin^2 x = \sin x \cos x$ for $0 \leq x < 2\pi$. Give your answers to 3 significant figures. [5 marks]

Answer .....

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C1 Mia solves $\sin 2x = 0.6$ for $0^\circ \leq x < 360^\circ$. She writes $2x = 36.9^\circ$ or 143.1°, so $x = 18.4^\circ$ or 71.6°. Explain what she has missed, and give all the solutions to 1 decimal place. [4 marks]

Answer .....

C2 Solve $3\sec^2 x - 5\tan x - 5 = 0$ for $0^\circ \leq x < 360^\circ$. Give your answers to 1 decimal place. [5 marks]

Answer .....

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C3 Solve $\cos(x + 60^\circ) = \sin x$ for $0^\circ \leq x < 360^\circ$. Show that $\tan x = 2 - \sqrt{3}$ and hence give exact answers. [4 marks]

Answer .....

C4 Solve $\cot x = 2\cos x$ for $0^\circ < x < 360^\circ$. [4 marks]

Answer .....

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C5 Solve $\operatorname{cosec} 2x = 4$ for $0 \leq x < \pi$. Give your answers to 3 significant figures. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1

(a) Show that $2\sin(x + 30°) = \sqrt{3}\sin x + \cos x$. [2 marks]

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(b) Hence solve $2\sin(x + 30^\circ) = 3\cos x$ for $0^\circ \leq x < 360^\circ$, giving your answers to 1 decimal place. [4 marks]

Answer .....

(c) Hence solve $2\sin(2\theta + 30^\circ) = 3\cos 2\theta$ for $0^\circ \leq \theta < 180^\circ$, giving your answers to 1 decimal place. [3 marks]

Answer .....

Total for D1: 9 marks

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D2

> Sam writes: "$\sec^2 x = \tan^2 x - 1$, so $\tan^2 x - 2\tan x - 5 = 0$ and $\tan x = 1 \pm \sqrt{6}$."

Sam is asked to solve $\sec^2 x = 2\tan x + 4$ for $0° \leq x < 360°$.

(a) Explain the mistake in Sam's working. [2 marks]

(b) Solve the equation correctly, giving your answers to 1 decimal place where needed. [6 marks]

Answer .....

Total for D2: 8 marks

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D3 Solve $\cos(x + 45^\circ) + \cos(x - 45^\circ) = \sin 2x$ for $0^\circ \leq x < 360^\circ$. [7 marks]
Give exact answers.

Answer .....

D4

(a) Show that $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x} = 2\sec^2 x$, for $\sin x \neq \pm 1$. [3 marks]

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(b) Hence solve $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=8$ for $0 \leq x < 2\pi$, giving exact answers. [4 marks]

Answer .....

(c) Explain why $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=1$ has no solutions. [2 marks]

Total for D4: 9 marks

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D5

(a) Show that the equation $\tan(x + 45^\circ) = k \tan x$, where k is a constant, can be written as $k \tan^2 x + (1 - k) \tan x + 1 = 0$. [3 marks]

(b) Solve $\tan(x + 45^\circ) = 6 \tan x$ for $0^\circ \leq x < 360^\circ$, giving your answers to 1 decimal place. [4 marks]

Answer .....

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(c) Find the set of values of k for which $\tan(x + 45^\circ) = k \tan x$ has solutions. Give your answer in exact form. [3 marks]

Answer .....

Total for D5: 10 marks

D6

(a) Show that $\sin(x + 60^\circ) + \sin(x - 60^\circ) = \sin x$. [2 marks]

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(b) Hence solve $\sin(x + 60^\circ) + \sin(x - 60^\circ) = 2\cos^2 x$ for $0^\circ \leq x < 360^\circ$, giving your answers to 1 decimal place. [4 marks]

Answer .....

(c) Hence solve $\sin(2\theta + 60^\circ) + \sin(2\theta - 60^\circ) = 2\cos^2 2\theta$ for $0^\circ \leq \theta < 360^\circ$, giving your answers to 1 decimal place. [3 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Solve $\sin x + \sin 2x + \sin 3x = 0$ for $0 \leq x < 2\pi$. Give exact answers. [8 marks]

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Leave $\sin 2x$ alone. Write $\sin 3x$ as $\sin(2x + x)$ and $\sin x$ as $\sin(2x - x)$.

Hint 2 Add the two expansions: the $\cos 2x \sin x$ terms cancel, leaving $2 \sin 2x \cos x$.

Hint 3 Now $\sin 2x$ is a common factor. Do not divide by it: it can be 0.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can expand a compound angle and collect the terms into tan x = k.	○ ○ ○
I can replace cos 2x or sec²x so that the equation has one function, and solve the quadratic.	○ ○ ○
I can solve an equation in 2x or (2x + 30°) by stretching the interval first.	○ ○ ○
I can factorise instead of dividing, so that no solution is lost.	○ ○ ○

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