



## Solving trigonometric equations

71 marks

Date \_\_\_\_\_

### **A** Do Now

Quick questions to start.

- A1** [Last lesson] [2 marks] Simplify  $\frac{1 - \cos^2 2x}{\sin 2x \cos 2x}$ .
- A2** [Needed today] [2 marks] Write down the two angles between  $0^\circ$  and  $360^\circ$  whose cosine is  $-\frac{1}{2}$ .

Answer .....

Answer .....

- A3** [Needed today] [2 marks] The angle  $\theta$  is in the interval  $0^\circ \leq \theta \leq 360^\circ$ . Write down the interval that  $2\theta - 40^\circ$  is in.
- A4** [From chapter 9] [2 marks] In a triangle,  $\sin B = 0.42$  and angle  $B$  is obtuse. Find angle  $B$  to 1 decimal place.

Answer .....

Answer .....

## B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it. Give angles to 1 decimal place.

### EXAMPLE 1

Solve  $4 \cos \theta = -3$  for  $0^\circ \leq \theta < 360^\circ$ .

**Hint** Make  $\cos \theta$  the subject. The graph of  $y = \cos \theta$  is symmetrical about  $180^\circ$ , so the second value is  $360^\circ$  minus the first.

**B1** Solve  $9 \sin \theta + 4 = 0$  for [3 marks]  
 $-180^\circ \leq \theta \leq 180^\circ$ .

Answer  $\theta = \dots\dots$

### EXAMPLE 2

Solve  $3 \sin \theta = 7 \cos \theta$  for  
 $0^\circ \leq \theta < 360^\circ$ .

**Hint** Divide both sides by  $3 \cos \theta$  to get an equation in  $\tan \theta$ . Tan repeats every  $180^\circ$ .

**B2** Solve [3 marks]  
 $5 \sin x + 3 \cos x = 0$  for  
 $0^\circ \leq x < 360^\circ$ .

Answer  $x = \dots\dots$

### EXAMPLE 3

Solve  $\cos 2\theta = 0.3$  for  $0^\circ \leq \theta < 360^\circ$ .  
Give your answers to 1 decimal place.

**Hint** Here  $0^\circ \leq 2\theta < 720^\circ$ . Find every value of  $2\theta$  in that interval first, then halve them.

**B3** Solve  $\tan 3x = -1.8$  for [4 marks]  
 $0^\circ \leq x \leq 180^\circ$ .

Answer  $x = \dots\dots$

**EXAMPLE 4**

Solve  $\sin(\theta + 35^\circ) = -0.6$  for  
 $0^\circ \leq \theta < 360^\circ$ .

**Hint** Let  $u = \theta + 35^\circ$ , so  $35^\circ \leq u < 395^\circ$ .  
Solve for  $u$ , then subtract  $35^\circ$ .

**B4**

Solve

[4 marks]

$2 \tan(\theta - 25^\circ) = 5$  for  
 $-180^\circ \leq \theta \leq 180^\circ$ .

Answer  $\theta = \dots\dots$

**c Practice**

A-Level

**C1** Solve  $8 \sin^2 \theta = 5$  for  $-180^\circ \leq \theta \leq 180^\circ$ .

[4 marks]

Answer  $\theta = \dots\dots$

**C2** Find the exact solutions of  $2 \sin(2\theta + 30^\circ) = \sqrt{3}$  for  $0^\circ \leq \theta \leq 180^\circ$ . [4 marks]

Answer  $\theta = \dots\dots$

**C3** Solve  $\sin^2 x = 6 \cos^2 x$  for  $0^\circ \leq x < 360^\circ$ .

[4 marks]

Answer  $x = \dots\dots$

**C4** Without solving them, state how many solutions each equation has for  $0^\circ \leq x < 360^\circ$ . [3 marks]  
(a)  $\sin 4x = 0.9$     (b)  $\tan \frac{x}{2} = 3$     (c)  $2 \cos 3x = 3$

Answer (a) ...    (b) ...    (c) ...

## **D** Exam-style questions

Edexcel exam style

**D1**

Kai is asked to solve  $2 \cos 3\theta = \sqrt{2}$  for  $0^\circ \leq \theta \leq 180^\circ$ . Kai writes: " $\cos 3\theta = \frac{\sqrt{2}}{2}$ , so  $3\theta = 45^\circ$  and  $\theta = 15^\circ$ . The other solution is  $360^\circ - 15^\circ = 345^\circ$ , which is too big. So  $\theta = 15^\circ$  is the only solution."

Kai has not found every solution.

**(a)** Describe two mistakes in Kai's working.

[2 marks]

(b) Find all the solutions. Give them exactly.

[4 marks]

Answer  $\theta = \dots\dots$

Total for D1: 6 marks

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**D2** The angle  $x$  is in the interval  $0^\circ \leq x < 360^\circ$ .

(a) Show that the equation  $5 \sin(2x - 20^\circ) = 2 \cos(2x - 20^\circ)$  can be written as  $\tan(2x - 20^\circ) = 0.4$ . [2 marks]

(b) Hence solve  $5 \sin(2x - 20^\circ) = 2 \cos(2x - 20^\circ)$  for  $0^\circ \leq x < 360^\circ$ , giving your answers to 1 decimal place. [4 marks]

Answer  $x = \dots\dots$

Total for D2: 6 marks

**D3** A seat on a big wheel moves round once every 9 minutes. Its height above the ground,  $h$  metres,  $t$  minutes after it leaves the bottom, is modelled by  $h = 21 - 18 \cos(40t)^\circ$ , for  $0 \leq t \leq 9$ .

- (a) Write down the greatest height of the seat, and find the value of  $t$  [2 marks]  
when it is first reached.

Answer ..... m     $t = \dots\dots$

- (b) Find the values of  $t$  when the seat is 28 m above the ground. [4 marks]  
Give your answers to 2 decimal places.

Answer  $t = \dots\dots$

- (c) Hence find, in minutes, how long the seat is more than 28 m [2 marks]  
above the ground in each turn. Give your answer to 2 decimal  
places.

Answer ..... minutes

Total for D3: 8 marks

**D4** The curve  $C$  has equation  $y = \cos(x + 30^\circ)$ , for  $0^\circ \leq x \leq 360^\circ$ .

- (a) Sketch  $C$ . Give the coordinates of the points where  $C$  meets the coordinate axes, and of its maximum and minimum points. [4 marks]

- (b) Hence state the number of solutions of  $\cos(x + 30^\circ) = -0.35$  for  $0^\circ \leq x \leq 360^\circ$ . [1 mark]

Answer .....

- (c) Solve  $\cos(x + 30^\circ) = -0.35$  for  $0^\circ \leq x \leq 360^\circ$ , giving your answers to 1 decimal place. [3 marks]

Answer  $x = \dots\dots$

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Find every solution of  $\cos 5\theta = \cos \theta$  in the interval  $0^\circ \leq \theta \leq 180^\circ$ . [6 marks]  
Give your answers exactly.

Answer  $\theta = \dots\dots$

**Extension hints**

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Two angles have the same cosine when they are the same angle, or minus each other, give or take whole turns of  $360^\circ$ .
- Hint 2** So  $5\theta = \theta + 360^\circ n$  or  $5\theta = -\theta + 360^\circ n$ , where  $n$  is a whole number.
- Hint 3** The first gives  $\theta = 90^\circ n$ :  $0^\circ, 90^\circ, 180^\circ$ . The second gives  $\theta = 60^\circ n$ :  $0^\circ, 60^\circ, 120^\circ, 180^\circ$ . Together there are five different solutions.

**How did it go?**

Colour one circle on each line: not yet, nearly, yes.

I can solve  $\sin$ ,  $\cos$  or  $\tan$  of an angle equals a number in any given interval. ☐ ☐ ☒

I can turn a  $\sin = b \cos$  into an equation in  $\tan$ . ☐ ☐ ☒

I can change the interval first for a multiple or a shifted angle. ☐ ☐ ☒

I can say how many solutions an equation has, and explain why. ☐ ☐ ☒