

Stationary points and gradient functions

75 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-stationary-points-and-gradient/

Date _____

A Do Now

Quick questions to start.

A1 Needed today [2 marks] Solve $3x^2 - 12x - 15 = 0.$	A2 Last lesson [2 marks] Given that $y = 2x^3 - 9x^2 + 7, \text{ find } \frac{d^2y}{dx^2}.$
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Answer

Answer $\frac{d^2y}{dx^2} = \dots\dots$

A3 From chapter 7 [2 marks] Factorise fully $x^3 - 2x^2 - 5x + 6.$	A4 From chapter 2 [2 marks] Find the range of $f(x) = x^2 - 6x + 11.$
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Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the coordinates of the stationary points on the curve

$y = 2x^3 + 3x^2 - 36x + 5$, and determine their nature.

Hint Solve $\frac{dy}{dx} = 0$. Then put each x into $\frac{d^2y}{dx^2}$: negative means a maximum, positive a minimum.

B1 Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 - 24x + 10$, and determine their nature. [5 marks]

EXAMPLE 2

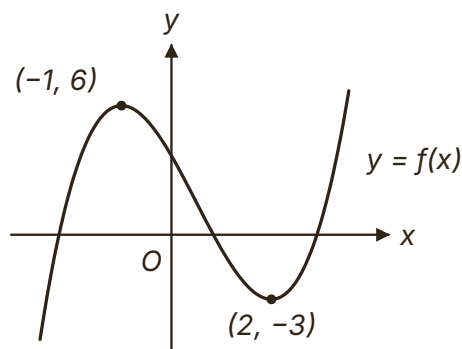
Find the stationary points on the curve $y = x^4 - 4x^3 + 6$, and determine their nature.

Hint At one point $\frac{d^2y}{dx^2} = 0$, which tells you nothing. Look at the sign of $\frac{dy}{dx}$ just either side.

B2 Find the stationary points on the curve $y = 3x^4 + 8x^3 - 1$, and determine their nature. [6 marks]

EXAMPLE 3

The diagram shows the cubic $y = f(x)$, with turning points $(-1, 6)$ and $(2, -3)$. For $y = f'(x)$, find where it meets the x -axis, where $f'(x) < 0$, and the x -coordinate of its turning point.

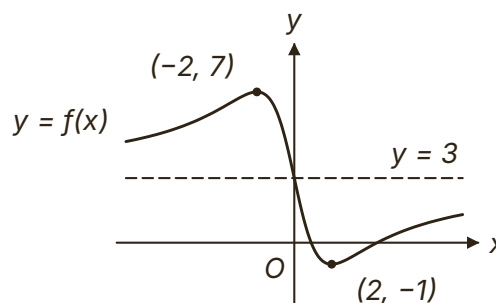


Not drawn accurately

Hint A turning point of $y = f(x)$ is where $f'(x) = 0$. Where f goes down, $f'(x)$ is below the x -axis.

B3 [4 marks]

The diagram shows $y = f(x)$, with turning points $(-2, 7)$ and $(2, -1)$ and the asymptote $y = 3$. For $y = f'(x)$, find where it meets the x -axis, its horizontal asymptote, and where $f'(x) > 0$.



Not drawn accurately

c Practice

A-Level

- C1** $f(x) = 5 + 32x - x^4$. Find the greatest value of $f(x)$, and hence state [4 marks]
the range of f .

Answer greatest value ; range

- C2** Find the coordinates of the stationary points on the curve $y = x(x - 6)^2$, and determine their nature. [5 marks]

- C3** The curve $y = x^3 + kx^2 + 3x$, where k is a constant, has no stationary points. Find the set of possible values of k . [3 marks]

Answer

D **Exam-style questions** Edexcel exam style

- D1** $f(x) = x^4 - 2x^3 - 3x^2 + 4x + 1$. You are given that $f'(2) = 0$.
- (a) Find the coordinates of the three stationary points of the curve $y = f(x)$. [4 marks]

(b) Determine the nature of each stationary point.

[3 marks]

(c) Find the set of values of k for which the equation $f(x) = k$ has exactly four real solutions.

[2 marks]

Answer

Total for D1: 9 marks

D2

The curve $y = x^5 - 5x^4 + 3$ has a stationary point at $(0, 3)$. Jo writes: " $\frac{d^2y}{dx^2} = 0$ at $x = 0$, so $(0, 3)$ is a point of inflection."

Jo's conclusion is wrong.

(a) Find the coordinates of both stationary points of the curve.

[4 marks]

Answer (....., ...) and (....., ...)

(b) Use $\frac{d^2y}{dx^2}$ to find the nature of the stationary point where $x = 4$.

[2 marks]

Answer $(4, -253)$ is a

- (c) Show that $(0, 3)$ is a maximum, and so explain why Jo is wrong. [3 marks]

Total for D2: 9 marks

D3 The curve C has equation $y = x + \frac{32}{x^2}$, $x > 0$.

- (a) Find the coordinates of the stationary point of C . [4 marks]

Answer (....., ...)

- (b) Determine the nature of the stationary point. [2 marks]

Answer

- (c) State the range of values of y on C . [1 mark]

Answer

Total for D3: 7 marks

D4 The curve $y = ax^3 + bx^2 - 12x + 4$, where a and b are constants, has a stationary point at $(1, -3)$.

(a) Find the value of a and the value of b .

[4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

(b) Find the coordinates of the other stationary point.

[3 marks]

Answer $(\dots\dots, \dots\dots)$

(c) Determine the nature of each stationary point.

[2 marks]

Answer $(1, -3)$ is a $\dots\dots$; $(-2, 24)$ is a $\dots\dots$

Total for D4: 9 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The curve $y = x^3 - 3kx^2 + 3(k^2 - 1)x$ depends on the constant k . [6 marks]
 Show that, for every value of k , the curve has a maximum and a minimum whose x -coordinates differ by 2. Find the difference between the y -coordinates of the maximum and the minimum.

Answer difference = ...

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Factor 3 out of $\frac{dy}{dx}$ and complete the square in x . Write it in terms of $(x - k)$.
- Hint 2** $\frac{dy}{dx} = 3((x - k)^2 - 1)$, so the stationary points are at $x = k - 1$ and $x = k + 1$.
- Hint 3** Put $x = k + u$. Then $y = u^3 - 3u + (k^3 - 3k)$. The stationary points are at $u = \pm 1$, and the last bracket is the same for both.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find stationary points and decide whether each is a maximum or a minimum. ☐ ☐ ☐

I can spot a point of inflection when the second derivative is zero. ☐ ☐ ☐

I can use a greatest or least value to find a range. ☐ ☐ ☐

I can sketch a gradient function from a curve. ☐ ☐ ☐