The binomial distribution

91 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-binomial-distribution/

Date

A Do Now

Quick questions to start. Some use earlier work; some set a slip that the examiner reports name.

A1 Needed today [2 marks]

Evaluate $\binom{12}{4}$, the number of ways of choosing 4 of 12 positions.

Answer .....

A2 Last lesson [3 marks]

The discrete random variable X has $\mathrm{P}(X = x) = kx$ for $x = 2, 3, 4, 5$. Find k and $\mathrm{P}(X \geqslant 4)$.

Answer .....

A3 Needed today [3 marks]

Each time Rae takes a penalty, she scores with probability 0.8, independently. She takes 3 penalties. Find the probability that she misses exactly one.

Answer .....

A4 Common slip [2 marks]

A calculator shows $0.019\,742\,7$. Write this probability correct to 3 significant figures and correct to 3 decimal places.

Answer .....

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PART 1 The binomial model and P(X = r)

EXAMPLE 1 · I DO

A courier finds that 12% of parcels arrive damaged. A shop receives 25 parcels. Define a suitable random variable, give its distribution with two conditions in context, and find the probability that exactly 3 parcels are damaged.

Hint Name the random variable in words first. The conditions must mention parcels, not just "trials".

YOUR TURN · WE DO

B1 A packet holds 30 seeds. Each seed fails to grow with probability 0.08. Define a suitable random variable, give its distribution with two conditions in context, and find the probability that exactly 2 seeds fail to grow. [5 marks]

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EXAMPLE 2 · I DO

For each situation, say whether X can be modelled by a binomial distribution. Give a reason. (i) A bag holds 5 red and 5 blue beads. Four beads are taken without replacement; X is the number of red beads. (ii) A fair coin is tossed until it shows a head; X is the number of tosses.

Hint Check each condition in turn: a fixed number of trials, two outcomes, independent trials, the same probability each time.

YOUR TURN · WE DO

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PRACTICE · YOU DO

C1 $X \sim \mathrm{B}(8, 0.3)$. Kai writes $\mathrm{P}(X = 2) = 0.3^2 \times 0.7^6 = 0.0106$. Explain his mistake and find $\mathrm{P}(X = 2)$. [4 marks]

B2 For each situation, say whether X can be modelled by a binomial distribution, giving a reason. If it can, state the distribution. (i) A fair six-sided die is rolled 10 times; X is the number of sixes. (ii) Five students are chosen at random from a class of 12 girls and 8 boys; X is the number of girls chosen. (iii) A train is late on any day with probability 0.15; X is the number of days in a week of 7 days on which it is late. [6 marks]

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C2 In a town, 35% of adults own a dog. A random sample of 40 adults is taken and X is the number who own a dog. Zara gives the conditions for $X \sim \mathrm{B}(40, 0.35)$ as "a fixed number of trials and two outcomes". Explain why her answer scores no marks, give two conditions in context, and find $\mathrm{P}(X = 14)$. [5 marks]

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PART 2 Cumulative probabilities and a second binomial

EXAMPLE 3 · I DO

$X \sim \mathrm{B}(15, 0.1)$. Find $\mathrm{P}(X < 2)$ and $\mathrm{P}(X \geqslant 1)$.

Hint "Less than 2" means 0 or 1. "At least 1" is everything except 0.

YOUR TURN · WE DO

B3 $X \sim \mathrm{B}(20, 0.15)$. Find $\mathrm{P}(X < 2)$, $\mathrm{P}(X \geqslant 1)$ and $\mathrm{P}(X = 2)$. [6 marks]

Answer .....

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EXAMPLE 4 · I DO

Bolts are packed in bags of 10. Each bolt is faulty with probability 0.05, independently. A bag is rejected if it holds 2 or more faulty bolts. Find the probability that a bag is rejected. Six bags are checked. Find the probability that exactly one of them is rejected.

Hint First find $q = \mathrm{P}(\text{a bag is rejected})$. Then each bag is a trial with success probability q: use a new random variable.

YOUR TURN · WE DO

B4 Eggs are packed in boxes of 8. Each egg is cracked with probability 0.06, independently. A box is returned if it holds 2 or more cracked eggs. Five boxes are bought. Find the probability that exactly 2 boxes are returned, and the probability that at least one box is returned. [6 marks]

Answer .....

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C3 Mo models the number of rainy days in a week, R, by $R \sim \mathrm{B}(7, 0.3)$. Find the probability of at most one rainy day in a week. [3 marks]

Answer .....

C4 A station has 10 trains a day. Each train is late with probability 0.1, independently. A day is called "bad" if 3 or more trains are late. Ali says the probability of exactly 2 bad days in 20 days is $\binom{20}{2}(0.1)^2(0.9)^{18}$. Explain his mistake and find the correct probability. [6 marks]

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PART 3 Finding n or p

EXAMPLE 5 · I DO

(i) $X \sim \mathrm{B}(n, 0.2)$. Find the least n such that $\mathrm{P}(X = 0) < 0.05$. (ii) $Y \sim \mathrm{B}(4, p)$ and $\mathrm{P}(Y = 0) = 0.0256$. Find p.

Hint $\mathrm{P}(X = 0) = 0.8^n$. Take logarithms, or try values of n. Remember the inequality changes direction when you divide by $\ln 0.8 < 0$.

YOUR TURN · WE DO

B5 (i) $X \sim \mathrm{B}(n, 0.15)$. Find the least n such that $\mathrm{P}(X \geqslant 1) > 0.9$. (ii) $Y \sim \mathrm{B}(5, p)$ and $\mathrm{P}(Y = 5) = \dfrac{1}{32}$. Find p. [5 marks]

Answer .....

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C5 $X \sim \mathrm{B}(n, 0.3)$. Find the least n such that $\mathrm{P}(X \geqslant 1) > 0.95$. [3 marks]

Answer .....

C6 $X \sim \mathrm{B}(7, p)$ and $\mathrm{P}(X = 2) = \mathrm{P}(X = 3)$. Find p, and hence find $\mathrm{P}(X = 0)$. [5 marks]

Answer .....

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D1 A garden centre sells tulip bulbs in bags of 12. Each bulb fails to flower with probability 0.15.

(a) Using $X \sim \mathrm{B}(12, 0.15)$ for the number of bulbs in a bag that fail to flower, state two conditions in context. [2 marks]

(b) Find the probability that exactly 2 bulbs in a bag fail to flower. [2 marks]

Answer .....

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(c) Find the probability that fewer than 2 bulbs in a bag fail to flower. [3 marks]

Answer .....

(d) Five bags are bought. Find the probability that exactly 3 of them have fewer than 2 bulbs that fail to flower. [3 marks]

Answer .....

Total for D1: 10 marks

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D2

> Ed says: "$\mathrm{P}(X \geqslant 2) = 1 - \mathrm{P}(X = 1) = 0.7298$."

Drinking glasses are packed in boxes of 20. Each glass is chipped with probability 0.1, independently, and X is the number of chipped glasses in a box.

(a) Explain Ed's mistake. Find the probability that a box has at least 2 chipped glasses. [4 marks]

(b) Find the probability that a box has more than 2 but fewer than 5 chipped glasses. [3 marks]

Answer .....

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(c) Give a reason why the chips in one box might not be independent. [2 marks]

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $X \sim \mathrm{B}(n, p)$. Given that $\mathrm{P}(X = 1) = 3\,\mathrm{P}(X = 0)$ and $\mathrm{P}(X = 2) = 4.125\,\mathrm{P}(X = 0)$, find n and p, and find $\mathrm{P}(X \geqslant 2)$. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Divide $\mathrm{P}(X = 1)$ by $\mathrm{P}(X = 0)$. Most of the powers cancel. Write the result in terms of $\dfrac{p}{1-p}$.

Hint 2 Do the same with $\mathrm{P}(X = 2)$. You now have two equations in n and $\dfrac{p}{1-p}$.

Hint 3 From the first, $\dfrac{p}{1-p} = \dfrac{3}{n}$. Substitute into the second and solve a linear equation in n.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can state the conditions for a binomial model in the context of the question.	○ ○ ○
I can find $\mathrm{P}(X = r)$ for $X \sim \mathrm{B}(n, p)$ and probabilities such as $\mathrm{P}(X < 2)$ and $\mathrm{P}(X \geqslant 1)$.	○ ○ ○
I can use a second binomial distribution when each bag, box or day passes or fails with a probability I have found.	○ ○ ○
I can find n or p from a given probability.	○ ○ ○

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