Critical regions

98 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-critical-regions/

Date

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

A Do Now

Quick questions to start: the six steps from the last sheet, and two slips that matter today.

A1 Needed today [2 marks]

Here are the six steps of a hypothesis test, out of order: Compare; Model; Conclude; Level and tails; Hypotheses; Calculate assuming H_0. Write them in the right order, and say what Step 4 gave you on the last sheet.

A2 Needed today [3 marks]

$X \sim \mathrm{B}(20, 0.3)$. Find $\mathrm{P}(X \leqslant 2)$ and $\mathrm{P}(X \geqslant 10)$.

Answer .....

A3 Common slip [2 marks]

$X \sim \mathrm{B}(30, 0.2)$. Ali writes $\mathrm{P}(X \geqslant 9) = 1 - \mathrm{P}(X \leqslant 9) = 0.06109$. Explain his mistake and find the correct probability.

Answer .....

A4 Common slip [2 marks]

A test of $\mathrm{H}_0 : p = 0.3$ against $\mathrm{H}_1 : p > 0.3$, where p is the proportion of a town's adults who own an electric bike, gives a p-value of 0.0721. The level is 5%. Ben writes: "Reject H_0 and accept H_1. The proportion is more than 0.3." Write the Step 5 and Step 6 that Ben should have written.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

PART 1 Critical regions from a table

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 1 · I DO

A bus company says that 40% of the commuters in a town travel by bus. After a fare rise, a councillor believes that the proportion has gone down. She will ask a random sample of 30 commuters and test at the 5% level. The table gives some values of $\mathrm{P}(X \leqslant k)$ when $X \sim \mathrm{B}(30, 0.4)$. (a) Find the critical region. (b) Find the actual significance level. (c) 8 of the commuters travel by bus. Complete the test.

k	$\mathrm{P}(X \leq k)$
5	0.005659
6	0.01718
7	0.04352
8	0.09401
9	0.1763

Hint "Gone down": small values support H_1, so the critical region is at the BOTTOM: $X \leqslant c$. Read $\mathrm{P}(X \leqslant c)$ straight from the table and find the LARGEST c with $\mathrm{P}(X \leqslant c) < 0.05$.

YOUR TURN · WE DO

B1 A survey found that 30% of the members of a gym go at least three times a week. The manager believes the proportion is now lower. In a random sample of 40 members, 6 go at least three times a week. The table gives some values of $\mathrm{P}(X \leqslant k)$ when $X \sim \mathrm{B}(40, 0.3)$.

k	$\mathrm{P}(X \leq k)$
4	0.002561
5	0.008618
6	0.02376
7	0.05528
8	0.1110

(a) Steps 1, 2 and 3: the model, the hypotheses, the level and tails. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Step 4: the critical region and the actual significance level, from the table. The test is at the 5% level. [3 marks]

Answer .....

(c) Steps 5 and 6. [2 marks]

Total for B1: 7 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2 · I DO

A plant grower says that 20% of the seeds in a mix give white flowers. Nia believes that a new mix gives a higher proportion of white flowers. She grows 25 seeds from the new mix and counts X, the number that give white flowers. She will test at the 5% level. The table gives some values of $\mathrm{P}(X \leqslant k)$ when $X \sim \mathrm{B}(25, 0.2)$. (a) Find the critical region for the test. (b) Find the actual significance level. (c) 10 of her seeds give white flowers. Complete the test.

k	$\mathrm{P}(X \leq k)$
6	0.7800
7	0.8909
8	0.9532
9	0.9827
10	0.9944

Hint The critical region is the set of values of X that would make you reject H_0. For "higher" it is the TOP end: $X \geqslant c$, with $\mathrm{P}(X \geqslant c) < 0.05$. Use $\mathrm{P}(X \geqslant c) = 1 - \mathrm{P}(X \leqslant c - 1)$.

YOUR TURN · WE DO

B2 A council says that 35% of the households in a town have solar panels. A salesman believes that the proportion is higher. He asks a random sample of 20 households, and 11 have solar panels. Test his belief at the 5% level using a critical region, in the steps below. The table gives some values of $\mathrm{P}(X \leqslant k)$ when $X \sim \mathrm{B}(20, 0.35)$.

k	$\mathrm{P}(X \leq k)$
9	0.8782
10	0.9468
11	0.9804
12	0.9940
13	0.9985

(a) Step 1 Model: define X and give its distribution if H_0 is true. [1 mark]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Step 2 Hypotheses. [2 marks]

Answer .....

(c) Step 3 Level and tails: say which end of the distribution the critical region is at. [1 mark]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(d) Step 4 Calculate assuming H_0: use the table to find the critical region and the actual significance level. [3 marks]

Answer .....

(e) Step 5 Compare. [1 mark]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 A test of $\mathrm{H}_0 : p = 0.6$ against $\mathrm{H}_1 : p < 0.6$ uses $X \sim \mathrm{B}(25, 0.6)$ at the 5% level. The observed value is 11. Dev writes: "$\mathrm{P}(X \leqslant 10) = 0.03439$ and $\mathrm{P}(X \leqslant 11) = 0.07780$. The first value above 0.05 is at 11, so the critical region is $X \leqslant 11$. 11 is in the region, so I reject H_0." Explain Dev's mistake, give the correct critical region, and complete the test. [5 marks]

(f) Step 6 Concludes in context [2 marks]

Total for B2: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C2 A test of $\mathrm{H}_0 : p = 0.35$ against $\mathrm{H}_1 : p > 0.35$ uses $X \sim \mathrm{B}(60, 0.35)$ at the 5% level. Three students give their answers. Ana: "The critical region is $\mathrm{P}(X \geqslant 28) = 0.04116$." Ben: "$\mathrm{P}(X \geqslant 27) = 1 - \mathrm{P}(X \leqslant 27) = 0.04116 < 0.05$, so the critical region is $X \geqslant 27$." Cara: "The critical region is $X \geqslant 28$, and the actual significance level is 5%." Say what is wrong with each answer, and give the correct critical region and actual significance level. [6 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

PART 2 Critical regions on a calculator

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 3 · I DO

A shop says that 35% of its customers pay with a phone. A bank believes that the proportion is higher, and will check a random sample of 25 customers. Using a calculator, find the critical region for a test at (a) the 10% level, (b) the 1% level. Give the actual significance level each time.

Hint There is no table now. Work out $\mathrm{P}(X \geqslant c)$ for values of c near the top, and find the smallest c with $\mathrm{P}(X \geqslant c)$ below the level. Start near the mean $25 \times 0.35 = 8.75$ and move up.

YOUR TURN · WE DO

B3 A clinic says that 45% of its patients wait more than 20 minutes. After a change to the booking system, the manager believes this proportion is lower. She will look at a random sample of 35 patients and test at the 10% level.

(a) State the hypotheses and the distribution of the test statistic if H_0 is true. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Use a calculator to find the critical region and the actual significance level. [4 marks]

PRACTICE · YOU DO

C3 A council says that 25% of the streetlights in a city are faulty. After a repair scheme, an engineer believes that the proportion is lower. She will check a random sample of 50 streetlights. Find the critical region for a test at the 10% level, and its actual significance level. [3 marks]

Answer .....

Total for B3: 6 marks

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 A company says that 65% of its parcels are delivered the day after they are posted. A customer believes that the proportion is lower. Of a random sample of 40 parcels, 20 are delivered the next day. Use a critical region to test, at the 5% level of significance, whether there is evidence for the customer's belief. State your hypotheses clearly, and give the actual significance level of the test. [8 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

PART 3 Two-tailed critical regions

EXAMPLE 4 · I DO

A school says that 25% of its students walk to school. A teacher thinks this proportion has changed. She will ask a random sample of 30 students and carry out a two-tailed test at the 5% level. Find the critical region, with the probability in each tail less than 0.025, and the actual significance level.

Hint "Changed" can be up or down, so there are TWO regions, one at each end. Each tail gets at most half the level: 0.025. Find the bottom region and the top region separately.

YOUR TURN · WE DO

B4 A spinner has half of its sectors blue. Omar wants to test whether the spinner is biased. He will spin it 40 times and carry out a two-tailed test at the 5% level. Find the critical region, with the probability in each tail less than 0.025, and the actual significance level. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C5 A website finds that 30% of its visitors use a tablet. The owner believes that this proportion has changed. She takes a random sample of 50 visitors. Using a 5% level of significance, find the critical region. The probability in each tail should be as close as possible to 2.5%. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C6 A two-tailed test of $\mathrm{H}_0 : p = 0.4$ at the 5% level uses $X \sim \mathrm{B}(30, 0.4)$, with the probability in each tail less than 0.025. Eve writes: "The critical region is $7 \leqslant X \leqslant 17$ and the actual significance level is 0.02124. The observed value is 5, which is not in my region, so I accept H_0." Find the correct critical region and actual significance level, and explain what is wrong with each part of Eve's answer. [6 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D1

A shop finds that 30% of its customers buy something from the bakery counter. After moving the counter, the manager believes that the proportion is higher. In a random sample of 35 customers, 15 buy something from the bakery counter. Tom tests at the 5% level. He writes: "$X \sim \mathrm{B}(35, 0.3)$. $\mathrm{P}(X \geqslant 15) = 1 - \mathrm{P}(X \leqslant 15) = 0.03588$, which is less than 0.05. So the critical region is $X \geqslant 15$ and the actual significance level is 0.05. 15 is in the critical region, so there is evidence that the proportion is higher."

Tom's test is wrong.

(a) Identify two errors in Tom's working. [2 marks]

(b) Find the correct critical region and actual significance level. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Complete the test correctly. [4 marks]

Total for D1: 10 marks

D2 A charity says that 20% of the people it phones agree to give money. A new caller believes that she does better than this. In a random sample of 45 of her calls, 14 people agree to give money.

(a) Write down suitable hypotheses for a test of the caller's belief. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Using a 5% level of significance, find the critical region for the test. [3 marks]

Answer .....

(c) State the actual significance level of the test, and explain what it means in this context. [2 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(d) The caller says, "$\frac{14}{45} = 31\%$, which is far more than 20%, so I do better." Complete the test and comment on her statement. [3 marks]

Total for D2: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A test of $\mathrm{H}_0 : p = p_0$ against $\mathrm{H}_1 : p > p_0$ is carried out at the 5% level with a random sample of 20. The value of p_0 is given to 2 decimal places, and the critical region for the test is $X \geqslant 9$. Find all the possible values of p_0. Show clearly how you know that no other value is possible. [9 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 For the critical region to be exactly $X \geqslant 9$, two things must be true when $p = p_0$: $\mathrm{P}(X \geqslant 9) < 0.05$, AND 8 must NOT be in the region.

Hint 2 So you need $\mathrm{P}(X \geqslant 9) < 0.05$ and $\mathrm{P}(X \geqslant 8) \geqslant 0.05$, with $X \sim \mathrm{B}(20, p_0)$. Both probabilities grow as p_0 grows.

Hint 3 Try $p_0 = 0.20, 0.21, 0.22, \ldots$ on a calculator. Find where $\mathrm{P}(X \geqslant 8)$ first reaches 0.05, and where $\mathrm{P}(X \geqslant 9)$ first reaches 0.05.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can read a critical region for a one-tailed test from a table of cumulative probabilities.	○ ○ ○
I can find a critical region on a calculator, for one tail or two, at a given level.	○ ○ ○
I can find the actual significance level and say what it means.	○ ○ ○
I can carry out a test by checking whether the observed value is in the critical region.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.