Cumulative binomial probabilities

97 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-cumulative-binomial-probabilities/

Date

A Do Now

Quick questions to start. Some use earlier work; some set a slip that the examiner reports name.

A1 Last lesson [2 marks]

$X \sim \mathrm{B}(10, 0.3)$. Use the formula to find $\mathrm{P}(X = 4)$.

Answer .....

A2 Common slip [3 marks]

X takes whole-number values. Write each probability using only $\mathrm{P}(X \leqslant \ldots)$: (i) $\mathrm{P}(X < 7)$, (ii) $\mathrm{P}(X \geqslant 7)$, (iii) $\mathrm{P}(3 \leqslant X < 8)$.

Answer .....

A3 Needed today [2 marks]

For $X \sim \mathrm{B}(10, 0.3)$, the tables give $\mathrm{P}(X \leqslant 4) = 0.8497$ and $\mathrm{P}(X \leqslant 3) = 0.6496$. Find $\mathrm{P}(X = 4)$ and compare with A1.

Answer .....

A4 Last lesson [2 marks]

A basketball player takes 20 free throws. X is the number she scores, and $X \sim \mathrm{B}(20, 0.7)$ is proposed. State two conditions in context.

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PART 1 Writing probabilities as cumulative probabilities

EXAMPLE 1 · I DO

$X \sim \mathrm{B}(20, 0.35)$. Find $\mathrm{P}(X \leqslant 5)$, $\mathrm{P}(X \geqslant 9)$, $\mathrm{P}(X > 4)$ and $\mathrm{P}(X < 8)$.

Hint Write each one with $\leqslant$ before you look anything up. "$\geqslant 9$" is everything except 0 to 8.

YOUR TURN · WE DO

B1 $X \sim \mathrm{B}(25, 0.2)$. Find $\mathrm{P}(X \leqslant 3)$, $\mathrm{P}(X \geqslant 7)$, $\mathrm{P}(X > 8)$ and $\mathrm{P}(X < 5)$. [6 marks]

Answer .....

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EXAMPLE 2 · I DO

$X \sim \mathrm{B}(30, 0.4)$. Find $\mathrm{P}(8 < X \leqslant 14)$, $\mathrm{P}(10 \leqslant X < 16)$, and the probability that X is between 9 and 13 inclusive.

Hint Write down the whole numbers in each range first. Then take the cumulative value at the top end away from the cumulative value just below the bottom end.

YOUR TURN · WE DO

B2 $X \sim \mathrm{B}(40, 0.25)$. Find $\mathrm{P}(6 < X \leqslant 12)$, $\mathrm{P}(8 \leqslant X < 15)$, and the probability that X is between 6 and 11 inclusive. [6 marks]

Answer .....

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C1 $X \sim \mathrm{B}(12, 0.4)$. Pip writes $\mathrm{P}(X \geqslant 6) = 1 - \mathrm{P}(X \leqslant 6) = 1 - 0.8418 = 0.1582$. Explain his mistake and find $\mathrm{P}(X \geqslant 6)$. [4 marks]

C2 Each email sent to a firm is spam with probability 0.08. In a batch of 25 emails, X are spam. Lucy is asked for the probability that fewer than 3 are spam and the probability that more than 3 are spam. She gives 0.8649 and 0.1351. Explain her mistake and find both probabilities. [5 marks]

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C3 $X \sim \mathrm{B}(12, 0.5)$. Explain, without calculating, why $\mathrm{P}(X \leqslant 3) = \mathrm{P}(X \geqslant 9)$. Find $\mathrm{P}(X \geqslant 9)$ and $\mathrm{P}(4 \leqslant X \leqslant 8)$. [5 marks]

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PART 2 The largest k and the smallest r

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EXAMPLE 3 · I DO

$X \sim \mathrm{B}(16, 0.3)$ is not in the printed tables, so part of its cumulative table is given. Find $\mathrm{P}(X = 5)$, $\mathrm{P}(X \geqslant 7)$ and $\mathrm{P}(3 < X \leqslant 8)$. Find the largest k with $\mathrm{P}(X \leqslant k) < 0.05$ and the smallest r with $\mathrm{P}(X \geqslant r) < 0.05$.

x	$\mathrm{P}(X \leq x)$
0	0.0033
1	0.0261
2	0.0994
3	0.2459
4	0.4499
5	0.6598
6	0.8247
7	0.9256
8	0.9743

Hint Every value in the table is a cumulative value. A single probability is a difference of two of them. For r, write $\mathrm{P}(X \geqslant r) = 1 - \mathrm{P}(X \leqslant r - 1)$ and try values.

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EXAMPLE 4 · I DO

$X \sim \mathrm{B}(50, 0.25)$. Find the largest k with $\mathrm{P}(X \leqslant k) < 0.05$, the largest k with $\mathrm{P}(X \leqslant k) < 0.025$, the smallest r with $\mathrm{P}(X \geqslant r) < 0.05$, and $\mathrm{P}(k < X < r)$ for the 5% values.

Hint Each answer needs two values: one below the limit and the next one above it. Show both.

YOUR TURN · WE DO

B3 [7 marks]

Part of the cumulative table for $X \sim \mathrm{B}(15, 0.35)$ is given. Find $\mathrm{P}(X = 6)$, $\mathrm{P}(X > 4)$ and $\mathrm{P}(2 \leqslant X < 8)$. Find the largest k with $\mathrm{P}(X \leqslant k) < 0.05$ and the smallest r with $\mathrm{P}(X \geqslant r) < 0.05$, showing the values you compare.

x	$\mathrm{P}(X \leqslant x)$
0	0.0016
1	0.0142
2	0.0617
3	0.1727
4	0.3519
5	0.5643
6	0.7548
7	0.8868
8	0.9578

B4 [6 marks]

$X \sim \mathrm{B}(60, 0.2)$. Use a calculator to find the largest k with $\mathrm{P}(X \leqslant k) < 0.025$ and the smallest r with $\mathrm{P}(X \geqslant r) < 0.025$, showing the values you compare. Find $\mathrm{P}(k < X < r)$.

Answer .....

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C4 A grower sells seeds in batches of 60. Each seed fails to germinate with probability 0.06, independently. X is the number of seeds in a batch that fail. The grower wants to print a number d on each packet so that the probability that more than d seeds fail is less than 0.01. Find the smallest such d. [3 marks]

Answer .....

Answer .....

C5 A firm wants the probability that a batch of 200 items has fewer than 6 faulty items to be at least 0.95. Each item is faulty with probability 0.015, independently. Ben calculates $\mathrm{P}(X = 6) = 0.0500$ and says "$1 - 0.05 = 0.95$, so the aim is met." Explain why Ben is wrong and decide whether the aim is met. [5 marks]

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PART 3 Finding the least n or p

EXAMPLE 5 · I DO

Each card in a pack of collector cards is a "gold" card with probability 0.1, independently. Find the least number of cards Fay must open so that the probability of at least 2 gold cards is more than 0.9.

Hint Write the probability for n cards as $1 - \mathrm{P}(X \leqslant 1)$. Try values of n and show the values either side of 0.9.

YOUR TURN · WE DO

B5 Each bolt is faulty with probability 0.08, independently. Find the least number of bolts that must be checked so that the probability of finding at least 2 faulty bolts is more than 0.8. [5 marks]

Answer .....

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C6 Seeds are sold in packets of 30. A seller wants the probability that at least 26 seeds in a packet grow to be at least 0.9. The seller can buy better seeds. Find, to 2 decimal places, the least probability p of a seed growing that meets the aim. [4 marks]

Answer .....

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C7 Each scratch card wins a prize with probability 0.15, independently. Find the least number of cards Dev must buy so that the probability of winning at least 2 prizes is more than 0.9. [6 marks]

Answer .....

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D1

> Lena says: "$\mathrm{P}(X \leqslant 14) = 0.0540$ is the value closest to 0.05, so $k = 14$."

$X \sim \mathrm{B}(50, 0.4)$. A teacher asks for the largest k with $\mathrm{P}(X \leqslant k) < 0.05$ and the smallest r with $\mathrm{P}(X \geqslant r) < 0.05$.

(a) Explain Lena's mistake and find k. [3 marks]

(b) Find r, showing the values you compare. [3 marks]

Answer .....

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(c) Find $\mathrm{P}(k < X < r)$. [3 marks]

Answer .....

Total for D1: 9 marks

D2 Apples are packed in bags of 20. Each apple is bruised with probability 0.1, independently. A bag is "grade A" if it holds at most 3 bruised apples.

(a) Find the probability that a bag is grade A. [2 marks]

Answer .....

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(b) A shop buys 12 bags. Find the probability that at least 10 of them are grade A. [4 marks]

Answer .....

(c) The 240 apples in the 12 bags are counted together. Find the probability that more than 30 of them are bruised. [3 marks]

Answer .....

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $X \sim \mathrm{B}(n, 0.2)$. Find the least value of n for which $\mathrm{P}(X \leqslant 2) < 0.05$. For this value of n, find the smallest r with $\mathrm{P}(X \geqslant r) < 0.05$, and find the probability that X lies in either of the two tails, $X \leqslant 2$ or $X \geqslant r$. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $\mathrm{P}(X \leqslant 2)$ gets smaller as n gets larger. Start near $n = 25$ and work up.

Hint 2 Show the values for the last n that fails and the first n that works.

Hint 3 For r, use $\mathrm{P}(X \geqslant r) = 1 - \mathrm{P}(X \leqslant r - 1)$ with $n = 30$. The two tails do not overlap, so their probabilities add.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can rewrite "fewer than", "at least" and "more than" as cumulative probabilities, using $\mathrm{P}(X \geqslant x) = 1 - \mathrm{P}(X \leqslant x - 1)$. ○ ○ ○

I can find $\mathrm{P}(a < X \leqslant b)$ from two cumulative values. ○ ○ ○

I can find the largest k with $\mathrm{P}(X \leqslant k) < 0.05$ and the smallest r with $\mathrm{P}(X \geqslant r) < 0.05$; these are the critical regions of hypothesis testing in chapter 7. ○ ○ ○

I can find the least n, or the least p, that meets a condition on a cumulative probability. ○ ○ ○

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