Hypothesis tests in exam questions

97 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-hypothesis-tests-in-exam-questions/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that the examiner reports say costs marks.

A1 Last lesson [2 marks]

A coin is thrown 20 times to test whether it is biased towards heads, and lands heads 15 times. Ali writes: "Let X be the number of heads. Under H_0, $X \sim \mathrm{B}(20, 0.5)$. $\mathrm{H}_0 : p = 0.5$, $\mathrm{H}_1 : p > 0.5$. $\mathrm{P}(X \geqslant 15) = 0.02069$. Reject H_0: there is evidence that the coin is biased towards heads." Which two of the six steps are missing?

Answer .....

A2 Common slip [2 marks]

A shop says that 15% of its customers pay with a voucher. Ben wants to test whether the proportion is higher, and writes $\mathrm{H}_0 : p = 0$, $\mathrm{H}_1 : p > 0.15$. Say what is wrong and write the hypotheses correctly.

Answer .....

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A3 Common slip [2 marks]

$X \sim \mathrm{B}(26, 0.15)$. Find $\mathrm{P}(X = 0)$, giving your answer to 4 significant figures.

Answer .....

A4 Last lesson [3 marks]

For each claim, say whether a test of it is one-tailed or two-tailed.
(i) The proportion of late trains has fallen.
(ii) A dice is biased.
(iii) The proportion of pupils who walk to school has changed.

Answer .....

A5 Last lesson [2 marks]

$X \sim \mathrm{B}(26, 0.35)$. You are given $\mathrm{P}(X \leqslant 4) = 0.02422$, $\mathrm{P}(X \leqslant 5) = 0.06492$, $\mathrm{P}(X \geqslant 13) = 0.08318$ and $\mathrm{P}(X \geqslant 14) = 0.03771$. Write down the critical region for a two-tailed test at the 10% level, with each tail at most 5%.

Answer .....

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PART 1 A test question in parts

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EXAMPLE 1 · I DO

A vaccine causes a mild reaction in 18% of patients. A new version of the vaccine is thought to cause fewer reactions. A doctor gives the new version to a random sample of 40 patients and records the number X who have a mild reaction.

(a) State two conditions needed for $\mathrm{B}(40, 0.18)$ to be a suitable model for X if the proportion has not changed.

(b) Find $\mathrm{P}(X = 3)$ for this model.

(c) Find the critical region for a test, at the 5% level of significance, of whether the proportion of patients with a reaction has fallen. State its actual significance level.

(d) 3 of the 40 patients have a reaction. Complete the test.

Hint Parts (c) and (d) are Steps 2 to 6. In (d), compare 3 with the critical region from (c), not $\mathrm{P}(X = 3)$ with 0.05.

YOUR TURN · WE DO

B1 A bus company says that 75% of its buses arrive on time. A passenger believes that the proportion is lower. She records whether each of a random sample of 28 buses arrives on time.

(a) State two conditions for the number of buses that arrive on time to be modelled by $\mathrm{B}(28, 0.75)$. Give them in context. [2 marks]

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(b) Write down the hypotheses and find the critical region for a test at the 5% level. State its actual significance level. [4 marks]

Answer .....

(c) 16 of the 28 buses arrive on time. Complete the test. [2 marks]

Answer .....

Total for B1: 8 marks

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C1 A student wants to test whether more than 30% of the cars that pass her school are electric. She records the next 20 cars that pass while she waits at the gate at 8.30 one morning, and plans to use $\mathrm{B}(20, 0.3)$. [4 marks]

(i) State the two conditions needed for a binomial model, in context.

(ii) Give one reason why her way of collecting the sample may not meet these conditions.

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C2 A seed packet says that 60% of the seeds germinate within a week. A gardener believes that a new compost increases this proportion. She sows 24 of the seeds in the new compost and 18 germinate within a week. She tests her belief at the 10% level. Do the test in three parts.

(a) Steps 1 and 2: define X, state its distribution under H_0, and write the hypotheses. [2 marks]

Answer .....

(b) Steps 3 and 4: state the level and tail, and find the probability. [2 marks]

Answer .....

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(c) Steps 5 and 6: compare and conclude. Would the conclusion be the same at the 5% level? [2 marks]

Answer .....

Total for C2: 6 marks

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PART 2 One-tailed and two-tailed tests

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EXAMPLE 2 · I DO

A cinema says that 25% of its tickets are bought at the door. The manager will record how 32 randomly chosen tickets were bought, to test whether the proportion bought at the door has changed, at the 5% level.
(a) Find the critical region if the probability in each tail is to be as close as possible to 2.5%. State its actual significance level.
(b) Find the critical region if the probability in each tail is to be at most 2.5%. State its actual significance level.

Hint Write down the tail probabilities on both sides of each boundary. "As close as possible" picks the nearer one, even if it is over 0.025; "at most" never goes over.

YOUR TURN · WE DO

B2 A gym says that 35% of its members go to at least one class each week. A manager will ask a random sample of 37 members, to test, at the 5% level, whether the proportion has changed.

(a) Find the critical region, with the probability in each tail as close as possible to 2.5%. [4 marks]

Answer .....

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(b) State the actual significance level and explain why it is more than 5%. [3 marks]

PRACTICE · YOU DO

C3 In a large town, 40% of households recycle glass. In a random sample of 35 households from one estate, 8 recycle glass. For $X \sim \mathrm{B}(35, 0.4)$, $\mathrm{P}(X \leqslant 8) = 0.02595$. For each test, say whether $\mathrm{H}_0 : p = 0.4$ is rejected. [5 marks]

(i) $\mathrm{H}_1 : p < 0.4$, at the 5% level.

(ii) $\mathrm{H}_1 : p \neq 0.4$, at the 5% level.

(iii) $\mathrm{H}_1 : p \neq 0.4$, at the 10% level.

Answer .....

Total for B2: 7 marks

Answer .....

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C4 A café says that 25% of its customers order a pastry. The owner believes that the proportion is lower on weekdays. She plans to record 45 randomly chosen weekday customers and to test at the 5% level, with the critical region $X \leqslant 6$. On the day, 5 of the 45 order a pastry. Raj says: "The critical region is $X \leqslant 6$, so the p-value of the test is $\mathrm{P}(X \leqslant 6) = 0.04461$." Explain Raj's mistake, find the p-value, and say what 0.04461 does measure. Complete the test. [5 marks]

Answer .....

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PART 3 Finding n or p

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EXAMPLE 3 · I DO

A survey found that 40% of commuters in a city listen to podcasts. A podcast company believes that the proportion has increased. In a random sample of 45 commuters, 24 listen to podcasts.
(a) Test the company's belief at the 5% level of significance.
(b) State the smallest significance level at which H_0 would be rejected.
(c) The company says, "This proves that the proportion has increased." Comment.

Hint Use the p-value: the probability of 24 or more. In (b), H_0 is rejected whenever the level is at least the p-value.

YOUR TURN · WE DO

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B3 In a town, 55% of voters support a plan for a new swimming pool. A councillor believes that the proportion of voters in her ward who support the plan is different. She asks a random sample of 36 voters in her ward, and 25 of them support the plan. Test, at the 10% level of significance, whether the councillor's belief is supported. [7 marks]

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EXAMPLE 4 · I DO

A basketball player scores with 40% of her long-range shots. After some training she takes n long-range shots and scores with 10 of them. A test of $\mathrm{H}_0 : p = 0.4$ against $\mathrm{H}_1 : p > 0.4$ is carried out at the 5% level, where p is the proportion of long-range shots she scores. Given that H_0 is rejected, find the possible values of n.

Hint For a fixed 10 successes, more shots make 10 less surprising. Find $\mathrm{P}(X \geqslant 10)$ for $X \sim \mathrm{B}(n, 0.4)$ as n increases, and see where it passes 0.05.

YOUR TURN · WE DO

B4 On a website, 15% of visitors leave a review. After a change, 6 of a random sample of n visitors leave a review. A test of $\mathrm{H}_0 : p = 0.15$ against $\mathrm{H}_1 : p > 0.15$ is carried out at the 5% level.

Answer

(a) Explain why a larger value of n makes 6 reviews less significant. [2 marks]

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(b) Given that H_0 is rejected, find the possible values of n. [4 marks]

PRACTICE · YOU DO

C5 At a company, 40% of job applicants are offered an interview. A new application form is introduced, and the manager believes that the proportion offered an interview has increased. Of a random sample of 48 applicants who used the new form, 27 are offered an interview. Find the smallest whole-number percentage significance level at which H_0 would be rejected. [2 marks]

Answer .....

Answer **Total for B4: 6 marks**

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C6 A machine is claimed to make a fault with probability p on each item. In a random sample of 15 items, none has a fault. A test of $\mathrm{H}_0 : p = p_0$ against $\mathrm{H}_1 : p \neq p_0$ is carried out at the 5% level, with each tail at most 2.5%. Find the values of p_0 for which H_0 is rejected. Give the boundary to 3 significant figures. [5 marks]

Answer .....

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D1

A council says that 65% of residents recycle their food waste. After a campaign, an officer tests, at the 5% level, whether the proportion has changed. In a random sample of 42 residents, 34 recycle their food waste. Tom writes:
"$\mathrm{H}_0 : X = 0.65$, $\mathrm{H}_1 : X > 0.65$. $\mathrm{P}(X = 34) = 0.01158$. $0.01158 < 0.05$, so accept H_1. The campaign has worked."

Tom's test has errors.

(a) Identify three errors in Tom's test. [3 marks]

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(b) Carry out the test correctly. [7 marks]

Answer .....

Total for D1: 10 marks

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D2 In a council area, 30% of residents use the local park at least once a week. After the park is renovated, the council asks a random sample of 44 residents whether they use the park at least once a week, to test whether the proportion has changed.

(a) State two assumptions needed to model the number of residents in the sample who use the park weekly by $\mathrm{B}(44, 0.3)$, if the proportion has not changed. Give them in context. [2 marks]

(b) Write down the hypotheses, and find the critical region for a test at the 5% level. The probability in each tail should be as close as possible to 2.5%. [4 marks]

Answer .....

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(c) State the actual significance level of this test. [2 marks]

Answer .....

(d) 7 of the 44 residents use the park weekly. Complete the test. [2 marks]

Answer .....

(e) Explain how the conclusion would change if each tail had to be at most 2.5%. [2 marks]

Total for D2: 12 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A spinner lands on red with probability 0.3. Each of 15 students spins it 25 times and, on their own results, tests $\mathrm{H}_0 : p = 0.3$ against $\mathrm{H}_1 : p > 0.3$ at the 5% level, using the critical region from $\mathrm{B}(25, 0.3)$. Find the probability that at least one of the students rejects H_0. One student did reject H_0 and says that the spinner is biased towards red. Comment. Find the smallest number of students for which the probability that at least one rejects H_0 is more than 0.5. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 First find the critical region and its actual significance level a for one student. That is the probability that one student rejects H_0 when it is true.

Hint 2 The students' results are independent, so the number who reject is $\mathrm{B}(15, a)$. "At least one" is $1 - \mathrm{P}(\text{none})$.

Hint 3 For the last part, solve $1 - (1 - a)^m > 0.5$ for the number of students m, using logarithms or trying values.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can answer a multi-part test question: conditions, critical region, actual significance level and conclusion.	○ ○ ○
I can choose one or two tails from the wording, and read whether each tail must be at most or as close as possible to half the level.	○ ○ ○
I can decide a result near the boundary, and find the smallest level at which the null hypothesis is rejected.	○ ○ ○
I can find the values of n, or of p, for which the null hypothesis would be rejected.	○ ○ ○

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