The language of hypothesis testing

104 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-language-of-hypothesis-testing/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that the examiners see every year.

A1 Needed today [3 marks]

$X \sim \mathrm{B}(24, 0.35)$. Find $\mathrm{P}(X \leqslant 4)$ and $\mathrm{P}(X \geqslant 13)$.

Answer .....

A2 Needed today [2 marks]

A council wants to know the proportion of households in a town that recycle glass. It asks 60 households and 21 say they do. State the population, and explain whether $\frac{21}{60} = 0.35$ is the proportion for the whole town.

A3 Common slip [3 marks]

Write each of these in terms of probabilities of the form $\mathrm{P}(X \leqslant k)$: (i) $\mathrm{P}(X < 6)$, (ii) $\mathrm{P}(X \geqslant 6)$, (iii) $\mathrm{P}(X > 6)$.

Answer .....

A4 Common slip [2 marks]

$X \sim \mathrm{B}(15, 0.25)$. Ruby is asked for the probability that X is at most 2. She writes $\mathrm{P}(X = 2) = 0.1559$. Explain her mistake and find the correct probability.

Answer .....

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PART 1 The test statistic and the hypotheses

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EXAMPLE 1 · I DO

A game uses a spinner with five equal sectors. One sector is gold. Leon thinks the spinner lands on gold more often than it should. He spins it 40 times and counts the number of times it lands on gold. (a) Describe the test statistic. (b) State the population parameter that is being tested. (c) Write down suitable null and alternative hypotheses.

Hint The test statistic is worked out from the sample. The hypotheses are about the probability of gold on ONE spin, not about the count.

YOUR TURN · WE DO

B1 A bakery says that 15% of its loaves are underweight. A trading standards officer believes that the proportion of underweight loaves is higher than this. She weighs a random sample of 30 loaves.

(a) Describe the test statistic, and give its distribution if the bakery is right. [2 marks]

Answer .....

(b) Write down suitable null and alternative hypotheses. [2 marks]

Answer .....

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(c) State whether the test is one-tailed or two-tailed. Give a reason. [1 mark]

Total for B1: 5 marks

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EXAMPLE 2 · I DO

For each claim, write down H_0 and H_1, say whether the test is one-tailed or two-tailed, and give the probability allowed in each tail of a test at the 5% level. (i) A factory says 8% of its bulbs are faulty; a new process is believed to have reduced this. (ii) A teacher thinks a coin is biased. (iii) A candidate believes her support is now more than the 40% found by an earlier poll.

Hint Read the claim: "reduced" or "decreased" gives $<$, "increased" or "more" gives $>$, "biased" or "changed" gives $\neq$. A two-tailed test splits the level between the two tails.

YOUR TURN · WE DO

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PRACTICE · YOU DO

C1 A phone company claims that 40% of its customers use its app. The manager of a shop believes that the proportion of the shop's customers who use the app is lower than this. She asks a random sample of 25 of the shop's customers, and 6 of them use the app. Write down two suitable hypotheses. [2 marks]

Answer .....

B2 For each claim, write down H_0 and H_1, say whether the test is one-tailed or two-tailed, and give the probability allowed in each tail of a test at the 10% level. (i) A manager believes that the proportion of late trains, 30% last year, has gone down. (ii) A player believes that the probability of a six on a dice has changed from $\frac{1}{6}$. (iii) A charity thinks that more than 65% of people now recognise its logo. [6 marks]

Answer .....

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C2 Each student below has written hypotheses for a test. Say what is wrong with each, and write correct hypotheses. (i) A dice is rolled 60 times to test whether it gives too many sixes. Sam writes $\mathrm{H}_0 : X = 10$, $\mathrm{H}_1 : X > 10$. (ii) In a sample of 40, 14 people own a dog; it is claimed that 30% of people own a dog, and Tia thinks it is more. Tia writes $\mathrm{H}_0 : p = 0.35$, $\mathrm{H}_1 : p > 0.35$. (iii) Uma tests whether a coin is biased and writes "H_0: the coin is fair, H_1: the coin lands heads more often". [6 marks]

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PART 2 The p-value and the decision

EXAMPLE 3 · I DO

A survey company says that 45% of adults in a city cycle to work at least once a week. A researcher believes that the proportion is lower. She asks a random sample of 30 adults and 8 say they do. She will test at the 5% level. (a) Explain the condition under which the null hypothesis would be rejected. (b) Find the p-value and say whether H_0 is rejected.

Hint The p-value is the probability, if H_0 is true, of the observed value or a value MORE EXTREME in the direction of H_1. Here H_1 is "lower", so more extreme means 8 or fewer.

YOUR TURN · WE DO

B3 A website says that 20% of its visitors buy something. After the site is redesigned, the manager believes the proportion is higher. In a random sample of 40 visitors, 13 buy something. The test is at the 5% level. Explain the condition under which the null hypothesis would be rejected, find the p-value, and say whether H_0 is rejected. [5 marks]

Answer .....

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C3 A delivery firm says 35% of its parcels arrive before noon. Kai believes fewer than 35% arrive before noon. In a random sample of 30 parcels, 6 arrive before noon. Kai writes: "$\mathrm{P}(X = 6) = 0.03531 < 0.05$, so reject H_0." Explain why Kai's method is wrong, and carry out the test correctly at the 5% level. [5 marks]

Answer .....

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C4 Three students tested $\mathrm{H}_0 : p = 0.5$ against $\mathrm{H}_1 : p > 0.5$, where p is the proportion of voters in a town who support a new park. Their conclusions are below. Say what is wrong with each, and write a correct conclusion. (i) p-value 0.21: "Accept H_0. Half the voters support the park." (ii) p-value 0.012: "This proves that more than half of the voters support the park." (iii) p-value 0.36: "Reject H_0. There is not sufficient evidence that more than half support the park." [6 marks]

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PART 3 A full hypothesis test

EXAMPLE 4 · I DO

A dentist finds that 25% of the children she sees need a filling. After a campaign about sugar, she believes this proportion has decreased. In a random sample of 40 children, 5 need a filling. Test her belief at the 5% level of significance.

Hint All six steps. Step 4 on the calculator: "5 or fewer" is $\mathrm{P}(X \leqslant 5)$.

YOUR TURN · WE DO

B4 A newsagent finds that 35% of customers buy a newspaper. She believes that this proportion has decreased. In a random sample of 50 customers, 12 buy a newspaper. Test her belief at the 5% level of significance. [6 marks]

Answer .....

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EXAMPLE 5 · I DO

A charity finds that 30% of people stopped in the street give money. A volunteer believes a new script will increase this proportion. She uses it on a random sample of 25 people. Test at the 5% level (a) if 11 people give money, (b) if 12 people give money. You are given that, if $p = 0.3$, $\mathrm{P}(X \geqslant 11) = 0.09780$ and $\mathrm{P}(X \geqslant 12) = 0.04425$.

Hint Steps 5 and 6. Compare the p-value with 0.05. Then write "reject" or "do not reject" AND one sentence about the PROPORTION in context. Do not say "proves" or "accept".

YOUR TURN · WE DO

B5 A garden centre says that 85% of its seeds grow into plants. A customer believes the proportion is lower. She sows a random sample of 40 seeds and 30 grow. She tests at the 5% level. You are given that, if $p = 0.85$, $\mathrm{P}(X \leqslant 30) = 0.06722$.

(a) Step 1 Model: define X and give its distribution if H_0 is true. [1 mark]

Answer .....

(b) Step 2 Hypotheses: write down H_0 and H_1. [2 marks]

Answer .....

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(c) Step 3 Level and tails. [1 mark]

Answer .....

(d) Step 5 Compare (step 4 is given above). [1 mark]

Answer .....

(e) Step 6 Conclude, in context. [2 marks]

Total for B5: 7 marks

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C5 A seed company states that 25% of its tulip bulbs give red flowers. A gardener believes that a new mix of bulbs gives a higher proportion of red flowers. She plants a random sample of 30 bulbs from the new mix and 13 give red flowers. Test, at the 5% level of significance, whether there is evidence to support the gardener's belief. State your hypotheses clearly. [8 marks]

Answer .....

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C6 A college says that 70% of its students pass their driving test first time. A driving instructor claims that her students do better. Of a random sample of 20 of her students, 18 pass first time. Ben says, "90% is more than 70%, so her students are better." Explain why Ben's reasoning is not enough, then test the instructor's claim at the 5% level in the steps below.

(a) Explain why Ben's reasoning is not enough. [1 mark]

(b) Step 1 Model and Step 2 Hypotheses. [3 marks]

Answer .....

(c) Step 3 Level and tails, and Step 4 Calculate assuming H_0. [3 marks]

Answer .....

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(d) Step 5 Compare and Step 6 Conclude. [2 marks]

Total for C6: 9 marks

D Exam-style questions

Edexcel exam style

D1

A shop says that 20% of its customers use a loyalty card. The owner believes the proportion is higher. In a random sample of 35 customers, 11 use a loyalty card. Leah writes: "$\mathrm{H}_0 : p = \frac{11}{35}$, $\mathrm{H}_1 : p > 0.2$. $X \sim \mathrm{B}(35, 0.2)$. $\mathrm{P}(X = 11) = 0.04035$, which is less than 0.05, so I accept H_1. This proves that more customers use a loyalty card."

Leah's test is wrong.

(a) Identify three errors in Leah's test. [3 marks]

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(b) Carry out the test correctly at the 5% level of significance. [7 marks]

Answer .....

Total for D1: 10 marks

D2 A farmer claims that 60% of the eggs from her hens hatch. A vet believes the proportion is lower. Of a random sample of 30 eggs, 14 hatch.

(a) State suitable hypotheses, defining the parameter you use. [2 marks]

Answer .....

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(b) Find the p-value, and explain in context what it measures. [2 marks]

Answer .....

(c) Carry out the test at the 10% level of significance. [2 marks]

(d) Carry out the test at the 5% level of significance. [2 marks]

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(e) The vet chooses the level after seeing the p-value. Explain why the level should be chosen before the data are collected. [2 marks]

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A quality manager will take a random sample of n items and test $\mathrm{H}_0 : p = 0.2$ against $\mathrm{H}_1 : p < 0.2$ at the 5% level, where p is the proportion of faulty items. Show that with a sample of 10 items the test can never lead to H_0 being rejected. Find the smallest n for which the test can reject H_0, and the smallest n for which exactly 1 faulty item would lead to H_0 being rejected. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 The most extreme result in the direction of H_1 is 0 faulty items. With $n = 10$, find $\mathrm{P}(X = 0)$ and compare it with 0.05.

Hint 2 The test can reject H_0 only if $\mathrm{P}(X = 0) = 0.8^n < 0.05$. Solve this, with logarithms or by trying values of n.

Hint 3 For 1 faulty item the p-value is $\mathrm{P}(X \leqslant 1) = 0.8^n + 0.2n(0.8)^{n-1}$. Try values of n on a calculator until it is less than 0.05.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can describe the test statistic and write the null and alternative hypotheses in terms of the parameter p.	○ ○ ○
I can decide from the wording whether a test is one-tailed or two-tailed, and give the level in each tail.	○ ○ ○
I can say what a p-value is and compare it with the significance level.	○ ○ ○
I can write a conclusion in context that is not assertive.	○ ○ ○

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