Mutually exclusive and independent events

116 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-mutually-exclusive-and-independent-events/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Needed today [3 marks]

A Venn diagram of probabilities shows 0.25 in A only, 0.15 in A and B, 0.3 in B only and 0.3 outside both. Find $\mathrm{P}(A)$, $\mathrm{P}(B)$ and $\mathrm{P}(A) \times \mathrm{P}(B)$.

Answer .....

A2 Common slip [3 marks]

$\mathrm{P}(A) = 0.4$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \text{ and } B) = 0.24$. Tom says A and B are mutually exclusive because $\mathrm{P}(A) + \mathrm{P}(B) = 1$. Explain why Tom is wrong, and find $\mathrm{P}(A \text{ or } B)$.

Answer .....

A3 Needed today [2 marks]

A fair six-sided die is rolled. E is the event "the score is even" and F is the event "the score is more than 4". Find $\mathrm{P}(E \text{ and } F)$ by listing, and find $\mathrm{P}(E) \times \mathrm{P}(F)$.

Answer .....

A4 Needed today [2 marks]

C and D are mutually exclusive, with $\mathrm{P}(C) = 0.35$ and $\mathrm{P}(D) = 0.2$. Find $\mathrm{P}(C \text{ or } D)$ and the probability that neither C nor D happens.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Two fair six-sided dice are rolled. A is the event "the total is 7", B is the event "both dice show the same number" and C is the event "the total is 8". (a) Show that A and B are mutually exclusive. (b) Are B and C mutually exclusive? (c) Find $\mathrm{P}(A \text{ or } B)$.

Hint Mutually exclusive means the two events cannot happen together. Look for an outcome in both.

B1 [5 marks]

X and Y are mutually exclusive, and Y and Z are mutually exclusive. $\mathrm{P}(X) = 0.35$, $\mathrm{P}(Y) = 0.25$, $\mathrm{P}(Z) = 0.3$ and $\mathrm{P}(X \text{ and } Z) = 0.1$. Draw a Venn diagram. Find $\mathrm{P}(X \text{ or } Z)$, the probability that none of the three events happens, and the probability that exactly one of them happens.

Answer .....

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EXAMPLE 2

(i) A and B are independent, with $\mathrm{P}(A) = 0.4$ and $\mathrm{P}(B) = 0.35$. Find $\mathrm{P}(A \text{ and } B)$, $\mathrm{P}(A \text{ or } B)$ and $\mathrm{P}(A \text{ but not } B)$. (ii) C and D are independent, with $\mathrm{P}(C) = 0.6$ and $\mathrm{P}(C \text{ and } D) = 0.15$. Find $\mathrm{P}(D)$ and $\mathrm{P}(C \text{ or } D)$.

Hint For independent events, $\mathrm{P}(A \text{ and } B) = \mathrm{P}(A) \times \mathrm{P}(B)$. Use it forwards in (i) and backwards in (ii).

B2 [5 marks]

J and K are independent events with $\mathrm{P}(J) = 0.3$ and $\mathrm{P}(J \text{ or } K) = 0.65$. Find $\mathrm{P}(K)$, the probability that neither event happens, and the probability that exactly one of the two events happens.

Answer .....

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EXAMPLE 3

(i) Of 80 students, 30 have a cat, 24 have a dog and 9 have both. Determine whether "has a cat" and "has a dog" are independent. (ii) $\mathrm{P}(S) = 0.6$, $\mathrm{P}(T) = 0.45$ and $\mathrm{P}(S \text{ but not } T) = 0.33$. Show that S and T are independent, and find $\mathrm{P}(S \text{ or } T)$.

Hint Turn the counts into probabilities first. Then compare $\mathrm{P}(A \text{ and } B)$ with $\mathrm{P}(A) \times \mathrm{P}(B)$, and write a conclusion.

B3 The table shows how 200 students travel to college. A student is chosen at random. Determine whether the events "walks" and "is in Year 12" are independent. Give your conclusion in context. [5 marks]

	Walks	Other
Year 12	36	54
Year 13	50	60

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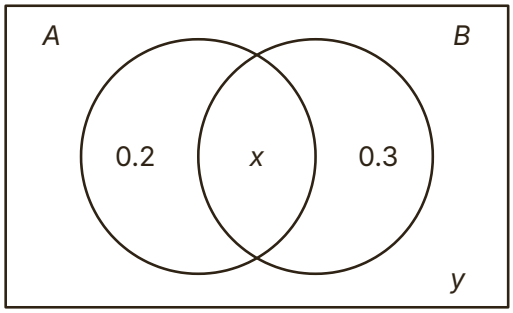
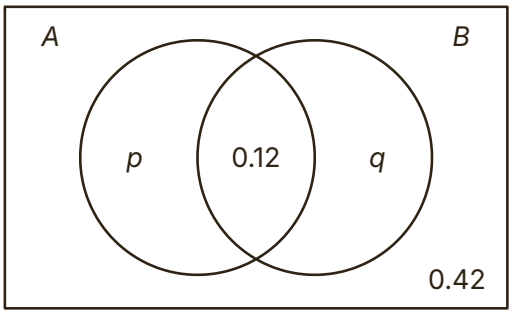
EXAMPLE 4

The Venn diagram shows the probabilities for two independent events A and B. Find the possible values of x and y.



Hint $\mathrm{P}(A) = 0.2 + x$, not 0.2. Independence gives $x = \mathrm{P}(A)\mathrm{P}(B)$, a quadratic in x. Then the four regions add to 1.

B4 The Venn diagram shows the probabilities for two independent events A and B, where $\mathrm{P}(A) < \mathrm{P}(B)$. Find p and q. [6 marks]



Answer .....

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EXAMPLE 5

In a class of 30 students, 12 play the guitar, 8 sing, and x do both. Show that "plays the guitar" and "sings" cannot be independent.

Hint If they were independent, $\frac{x}{30} = \frac{12}{30} \times \frac{8}{30}$. Solve for x and think about what x counts.

B5 In a club of 60 members, 25 play chess, 18 play bridge and x play both. (a) Show that "plays chess" and "plays bridge" cannot be independent. (b) Given that $x = 8$, find the probability that a member chosen at random plays neither game. [4 marks]

Answer .....

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C1 $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(A \text{ and } B) = 0.2$. Kai says that A and B are independent because $\mathrm{P}(A) + \mathrm{P}(B) = 1$. Explain why Kai's reason is wrong. Test whether A and B are independent, and find $\mathrm{P}(A \text{ or } B)$. [4 marks]

Answer .....

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C2 $\mathrm{P}(A) = 0.25$, $\mathrm{P}(B) = 0.4$, and A and B are mutually exclusive. Ruby says: "Mutually exclusive events cannot affect each other, so A and B are independent." Explain why Ruby is wrong, and find the probability that neither A nor B happens. [4 marks]

C3 Of 40 people, 10 own a bike, 16 own a car and 4 own both. Lee writes "$10 \times 16 = 160$, which is not 4, so owning a bike and owning a car are not independent." Explain Lee's mistake, and test correctly whether the two events are independent. [4 marks]

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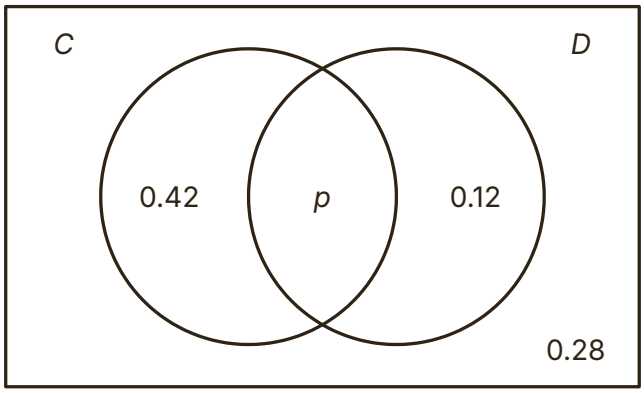
C4 The Venn diagram shows the probabilities for two events C and D. [4 marks]
Ali writes "$0.42 \times 0.12 = 0.0504$, which is not p, so C and D are not independent." Explain Ali's mistake. Find p and test whether C and D are independent.



Answer .....

C5 On a school day, R is the event "it rains" and L is the event "the bus is late". $\mathrm{P}(R) = 0.3$, $\mathrm{P}(L) = 0.2$ and $\mathrm{P}(R \text{ and } L) = 0.06$. Sara says that R and L are independent "because rain does not affect the bus". Show that R and L are independent using probabilities, and explain what this means in context. [4 marks]

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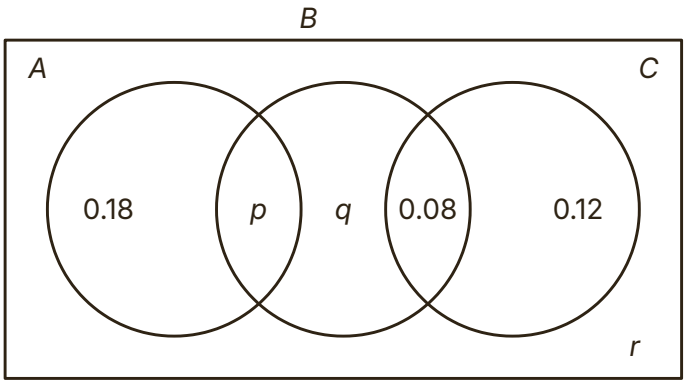
D1 The Venn diagram shows the probabilities for three events A, B and C. $\mathrm{P}(B) = 0.4$, and the events A and B are independent.

(a) Write down two of the events that are mutually exclusive, and give a reason. [2 marks]



(b) Find the value of p. [3 marks]

Answer .....

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(c) Find the values of q and r. [3 marks]

Answer .....

(d) Determine whether the events B and C are independent. [2 marks]

Answer .....

Total for D1: 10 marks

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D2 A sports club has 120 members. 45 play squash, x play tennis and 18 play both.

(a) Given that "plays squash" and "plays tennis" are independent, find x. [3 marks]

Answer .....

(b) Find the probability that a member chosen at random plays neither game. [2 marks]

Answer .....

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(c) 10 new members join. They all play tennis but not squash. Show that "plays squash" and "plays tennis" are now not independent. [4 marks]

Answer .....

Total for D2: 9 marks

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D3 The events A and B are independent, and $\mathrm{P}(A) = 2\,\mathrm{P}(B)$. The probability that exactly one of the events A and B happens is 0.44. Find $\mathrm{P}(A)$ and $\mathrm{P}(B)$, and the probability that neither event happens. [6 marks]

Answer .....

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D4 The table shows how the 250 students in a sixth form travel to school. A student is chosen at random.

	Bus	Car	Walk
Year 12	40	24	36
Year 13	60	46	44

(a) Show that the events "travels by bus" and "is in Year 12" are independent. [3 marks]

Answer .....

(b) Determine whether the events "travels by car" and "is in Year 12" are independent. Give your conclusion in context. [3 marks]

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(c) Next term, k of the Year 13 students who travel by car start to walk instead. Nothing else changes. Find k so that "walks" and "is in Year 12" are independent. [4 marks]

Answer .....

Total for D4: 10 marks

D5

> $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.3$, and A and B are independent. Jack says: "Because A and B are independent, $\mathrm{P}(A \text{ or } B) = \mathrm{P}(A) + \mathrm{P}(B) = 0.8$."

Jack is wrong.

(a) Explain Jack's mistake. [2 marks]

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(b) Find the correct value of $\mathrm{P}(A \text{ or } B)$. [2 marks]

Answer .....

(c) A third event C has $\mathrm{P}(C) = 0.25$. A and C are mutually exclusive, and B and C are independent. Find the probability that none of A, B and C happens. [5 marks]

Answer .....

Total for D5: 9 marks

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D6 $\mathrm{P}(S) = 0.35$, $\mathrm{P}(T) = 0.6$ and $\mathrm{P}(T \text{ but not } S) = 0.39$.

(a) Show that S and T are independent. [3 marks]

Answer .....

(b) Find $\mathrm{P}(S \text{ or } T)$. [2 marks]

Answer .....

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(c) Show that the events "T happens" and "S does not happen" are independent. [3 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 (a) Prove that if A and B are independent events, then A and "not B" are also independent. (b) A and B are independent. $\mathrm{P}(A \text{ but not } B) = 0.18$ and $\mathrm{P}(B \text{ but not } A) = 0.28$. Find the two possible pairs of values of $\mathrm{P}(A)$ and $\mathrm{P}(B)$. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 (a) $\mathrm{P}(A \text{ and not } B) = \mathrm{P}(A) - \mathrm{P}(A \text{ and } B)$. Replace $\mathrm{P}(A \text{ and } B)$ using independence and take out a factor of $\mathrm{P}(A)$.

Hint 2 (b) Let $\mathrm{P}(A) = a$ and $\mathrm{P}(B) = b$. By (a), $a(1-b) = 0.18$, and in the same way $b(1-a) = 0.28$.

Hint 3 Subtract the two equations: the ab terms cancel and give $b = a + 0.1$. Substitute into the first equation to get a quadratic in a.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use P(A or B) = P(A) + P(B) for mutually exclusive events.	○ ○ ○
I can use P(A and B) = P(A)P(B) for independent events, forwards and backwards.	○ ○ ○
I can test whether two events are independent and give the conclusion in context.	○ ○ ○
I can find unknown probabilities in a Venn diagram using independence.	○ ○ ○

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