One-tailed tests

101 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-one-tailed-tests/

Date

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A Do Now

Quick questions to start: the six steps, a critical region from the last sheet, and two slips.

A1 Needed today [2 marks]

Priya's test is set out below, with one of the six steps missing. Name the missing step, and write it. "A gym says 30% of members swim; Priya believes it is more. Model: X = number of 25 members who swim, $X \sim \mathrm{B}(25, 0.3)$. Level: 5%, one-tailed. Calculate: $\mathrm{P}(X \geqslant 12) = 0.04425$. Compare: $0.04425 < 0.05$. Conclude: reject H_0; there is sufficient evidence that the proportion who swim is more than 30%."

Answer .....

A2 Needed today [3 marks]

$X \sim \mathrm{B}(30, 0.2)$. A test of $\mathrm{H}_0 : p = 0.2$ against $\mathrm{H}_1 : p < 0.2$ is at the 5% level. Find the critical region and the actual significance level.

Answer .....

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A3 Common slip [2 marks]

A test of $\mathrm{H}_0 : p = 0.15$ against $\mathrm{H}_1 : p > 0.15$ uses $X \sim \mathrm{B}(20, 0.15)$, and the observed value is 6. Zak writes: "p-value $= 1 - \mathrm{P}(X \leqslant 6) = 0.02194$." Find the correct p-value and say whether H_0 is rejected at the 5% level.

Answer .....

A4 Common slip [2 marks]

A paper says 40% of adults in a city read the news online. Kim is asked to test whether this proportion has changed. In her sample, 55% read the news online, so she writes $\mathrm{H}_1 : p > 0.4$. Explain her mistake and write the correct H_1.

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PART 1 Upper-tail tests

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EXAMPLE 1 · I DO

A café finds that 30% of its customers order a hot chocolate. During a cold week, the manager believes this proportion is higher. In a random sample of 30 customers that week, 14 order a hot chocolate. Test, at the 5% level of significance, whether there is evidence for the manager's belief.

Hint Six steps. "Higher" makes the test one-tailed with the top tail, so the p-value is $\mathrm{P}(X \geqslant 14) = 1 - \mathrm{P}(X \leqslant 13)$.

YOUR TURN · WE DO

B1 A factory says that 15% of its phone cases have a scratch. A shop manager believes that the proportion is higher. In a random sample of 25 cases, 7 have a scratch. Test her belief at the 5% level of significance, in the steps below.

(a) Step 1 Model. [2 marks]

Answer .....

(b) Step 2 Hypotheses. [2 marks]

Answer .....

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(c) Step 3 Level and tails. [1 mark]

Answer .....

(d) Step 4 Calculate assuming H_0: find the p-value. [2 marks]

Answer .....

(e) Step 5 Compare. [1 mark]

Answer .....

(f) Step 6 Conclude, in context. [1 mark]

Total for B1: 9 marks

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EXAMPLE 2 · I DO

A survey says that 20% of adults in a region walk to work. A researcher believes the proportion is lower in one town. She asks a random sample of 60 adults there, and 10% of them walk to work. Test her belief at the 5% level of significance.

Hint The test statistic is a COUNT. Turn the percentage into a number of people first: 10% of 60.

YOUR TURN · WE DO

B2 A website says that 15% of its users click on an advert. A designer believes that a new layout gives a higher proportion. In a random sample of 48 users of the new layout, 25% click on the advert. Test the designer's belief at the 5% level of significance. [7 marks]

Answer .....

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C1 A firm says that 20% of its emails are opened within an hour. A manager believes the proportion is higher. In a random sample of 35 emails, 11 are opened within an hour. Leo writes: "$\mathrm{P}(X \geqslant 11) = 1 - \mathrm{P}(X \leqslant 11) = 0.03436 < 0.05$, so reject H_0." Explain Leo's mistake and carry out the test correctly at the 5% level. [5 marks]

Answer .....

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C2 A college says that 25% of its students cycle to college. The head of travel believes the proportion is higher. In a random sample of 40 students, 15 cycle. Test the belief at the 5% level, in the steps below.

(a) Step 1 Model and Step 2 Hypotheses. [3 marks]

Answer .....

(b) Step 3 Level and tails, and Step 4 Calculate assuming H_0. [3 marks]

Answer .....

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(c) Step 5 Compare and Step 6 Conclude. [2 marks]

Total for C2: 8 marks

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PART 2 Lower-tail tests

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EXAMPLE 3 · I DO

A town council says that 45% of residents use the town's recycling centre. A newspaper believes that the proportion is lower. In a random sample of 40 residents, 12 use the centre. Test the newspaper's belief at the 5% level of significance.

Hint "Lower" means the bottom tail: the p-value is $\mathrm{P}(X \leqslant 12)$, read straight from the calculator.

YOUR TURN · WE DO

B3 A charity says that 55% of the people who sign up to a fun run finish it. An organiser believes that, in wet weather, the proportion is lower. Of a random sample of 30 people who signed up for a wet run, 12 finished.

(a) Steps 1, 2 and 3. [3 marks]

Answer .....

(b) Step 4: the p-value. [2 marks]

Answer .....

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(c) Steps 5 and 6. [2 marks]

PRACTICE · YOU DO

C3 A test of $\mathrm{H}_0 : p = 0.4$ against $\mathrm{H}_1 : p < 0.4$ uses a random sample of 30, and the observed value is 15. Ola writes: "$\mathrm{P}(X \leqslant 15) = 0.1754$, which is more than 0.05, so I do not reject H_0." (a) Explain, without working out any probabilities, why this sample gives no evidence for H_1. (b) Write down the p-value Ola should have used, and state what it is. [5 marks]

Total for B3: 7 marks

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C4 A garden centre says that 70% of its bulbs flower in the first year. A customer believes the proportion is lower. Of a random sample of 25 bulbs, 14 flower in the first year. Test the customer's belief at the 5% level of significance. State your hypotheses clearly. [6 marks]

Answer .....

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PART 3 Conditions and the significance level

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EXAMPLE 4 · I DO

A hockey player scores 60% of her penalty shots. After extra training she believes she scores more often. She takes 35 penalty shots and scores 28. (a) State two conditions needed for a binomial model, in context, and say whether each is reasonable. (b) Test her belief at the 1% level. (c) Would the conclusion be different at the 5% level?

Hint The conditions are about each shot: is the chance of scoring the same every time, and does one shot affect the next? Then the same p-value is compared with each level.

YOUR TURN · WE DO

B4 An opinion poll found that 50% of voters in a town support a new road. After a public meeting, a campaigner believes that support has fallen. She asks a random sample of 28 voters, and 10 support the road.

(a) State two conditions needed for a binomial model, in context. [2 marks]

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(b) Test the campaigner's belief at the 10% level of significance. [5 marks]

PRACTICE · YOU DO

C5 A test of $\mathrm{H}_0 : p = 0.25$ against $\mathrm{H}_1 : p > 0.25$ uses a random sample of 20, and the observed value is 9. Find the p-value, and state whether H_0 is rejected at the 5% level and at the 1% level. [3 marks]

Answer .....

Answer .....

Total for B4: 7 marks

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C6 A shop says that 20% of customers pay by phone. Ash believes the proportion is higher. In a random sample of 50 customers, 16 pay by phone. Ash writes: "The sample proportion is $\frac{16}{50} = 0.32$, so $X \sim \mathrm{B}(50, 0.32)$. $\mathrm{P}(X \geqslant 16) = 0.5530 > 0.05$, so the number of customers paying by phone has not gone up." Explain the two mistakes in Ash's work, and complete the test correctly at the 5% level. [6 marks]

Answer .....

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D1

A cinema says that 15% of its customers buy a ticket online. The manager believes the proportion is now higher. In a random sample of 40 customers, 10 buy online. Jess writes: "$\mathrm{H}_0 : p = 0.15$, $\mathrm{H}_1 : p = 0.25$, because $\frac{10}{40} = 0.25$. $\mathrm{P}(X = 10) = 0.03730$, which is less than 0.05, so reject H_0. The proportion who buy online is now 25%."

Jess's test is wrong.

(a) Identify three errors in Jess's test. [3 marks]

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(b) Carry out the test correctly at the 5% level of significance. [7 marks]

Answer .....

Total for D1: 10 marks

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D2 A council says that 65% of the households in a district put out a food-waste bin each week. A councillor believes that, on one estate, the proportion is lower. She checks a random sample of 30 households on the estate, and 14 put out a food-waste bin.

(a) Give two conditions needed for a binomial model of the number of households, in context. Give one reason why one of them may not hold. [2 marks]

(b) Write down suitable hypotheses. [2 marks]

Answer .....

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(c) Test the councillor's belief at the 5% level of significance. [4 marks]

Answer .....

(d) State, with a reason, whether the conclusion would be the same at the 1% level. [2 marks]

Total for D2: 10 marks

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No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 An airline says that 90% of its flights land on time. A travel writer believes the proportion is lower. She tests this using a random sample of 40 of its flights, at the 5% level of significance. Ali sets up the test by counting the flights that are late. Sara sets it up by counting the flights that land on time. Write down each person's hypotheses, and show that the two tests have the same critical region when it is written as a number of flights that land on time. The writer's sample led to H_0 being rejected at the 5% level, but it would not have been rejected at the 1% level. Find every possible number of the 40 flights that landed on time. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Ali: Y = number of late flights, $Y \sim \mathrm{B}(40, 0.1)$ under H_0; the belief means Y is large. Sara: W = number on time, $W \sim \mathrm{B}(40, 0.9)$; the belief means W is small. $W = 40 - Y$.

Hint 2 Find the smallest c with $\mathrm{P}(Y \geqslant c) < 0.05$, and turn $Y \geqslant c$ into a statement about W.

Hint 3 Now find the critical region at the 1% level. The sample's value is in the 5% region but not in the 1% region.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can carry out a whole one-tailed test, upper or lower tail, in six steps.	○ ○ ○
I can find the p-value with the right tail and the right inequality.	○ ○ ○
I can state and criticise the conditions of the binomial model in context.	○ ○ ○
I can write a conclusion in context, and explain why a sample percentage alone is not evidence.	○ ○ ○

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