Calculating probabilities and Venn diagrams

124 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-probability-and-venn-diagrams/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Needed today [3 marks]

Two fair six-sided dice are rolled and the product of the two scores is recorded. Find the probability that the product is 12, and the probability that the product is greater than 20.

Answer .....

A2 Needed today [2 marks]

On a histogram, the bar for the class $15 \leqslant t < 25$ has frequency density 3.2. Find the frequency of this class. Another class, $25 \leqslant t < 40$, has frequency 24. Find the height of its bar.

Answer .....

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A3 Common slip [3 marks]

In a class of 32 students, 15 play football, 12 play tennis and 5 play both. Sam writes 15 in the "football only" region of a Venn diagram. Find the number who play football only, and the probability that a student chosen at random plays neither sport.

Answer .....

A4 Needed today [2 marks]

A grouped frequency table has the class $20 \leqslant x < 30$ with frequency 15. Estimate how many of these 15 values are less than 24.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A fair spinner has five equal sectors numbered 1 to 5. A second fair spinner has four equal sectors numbered 1 to 4. Both are spun and the positive difference between the two numbers is recorded (the difference is 0 if they are equal). Find the probability that (a) the difference is 1, (b) the difference is 2 or more.

Hint Draw a sample space diagram: a grid of 5×4 equally likely outcomes, each cell holding the difference. Count the cells.

B1 A fair four-sided die numbered 1 to 4 and a fair six-sided die are rolled, and the product of the two scores is recorded. Find the probability that the product is (a) odd, (b) 12 or more, (c) a square number. [4 marks]

Answer .....

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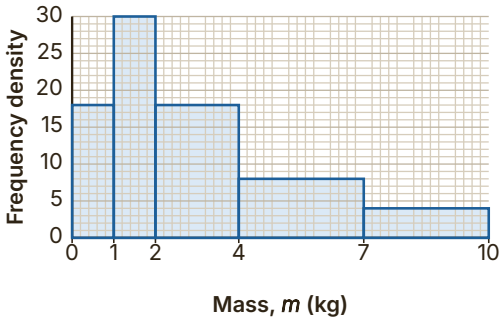
EXAMPLE 2

The table shows the lengths, in minutes, of 80 phone calls. A call is chosen at random. Find the probability that it lasted less than 5 minutes, and estimate the probability that it lasted more than 12 minutes. State the assumption you make.

Length, t (min)	Frequency
$0 \leq t < 2$	14
$2 \leq t < 5$	26
$5 \leq t < 10$	22
$10 \leq t < 15$	12
$15 \leq t < 25$	6

Hint Less than 5 minutes is two whole classes. More than 12 minutes cuts the class $10 \leqslant t < 15$: take the part of the class above 12 as a fraction of its width.

B2 The histogram shows the masses, in kg, of 120 parcels. A parcel is chosen at random. (a) Find the probability that its mass is more than 4 kg. (b) Estimate the probability that its mass is between 1.5 kg and 5 kg. [5 marks]



Answer .....

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EXAMPLE 3

In a group of 50 people, 28 have visited France, 19 have visited Spain and 9 have visited both. Draw a Venn diagram. A person is chosen at random. Find the probability that the person has visited (a) Spain but not France, (b) exactly one of the two countries, (c) neither country.

Hint Put the 9 in the overlap first. The 28 includes those 9, so France only is $28 - 9$. The four regions add to 50.

B3 A gym has 80 members. 34 use the pool, 21 use the pool but not the sauna, and 18 use neither. Draw a Venn diagram. A member is chosen at random. Find the probability that the member uses (a) the sauna, (b) exactly one of the two, (c) the pool or the sauna. [5 marks]

Answer .....

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EXAMPLE 4

A survey of 100 students asked whether they play hockey (H), netball (N) or go running (R). 6 do all three. 14 play hockey and netball, 11 play netball and run, and 9 play hockey and run. 38 play hockey, 45 play netball and 33 run. Draw a Venn diagram. A student is chosen at random. Find the probability that the student does (a) exactly one of the three, (b) at least two, (c) none, (d) netball but not hockey.

Hint Start in the centre with 6. Each pair total (such as 14) includes the 6, so subtract it. Then each single total minus the three regions inside it.

B4 [6 marks]

120 people were asked which of three streaming services X, Y and Z they use. 4 use all three. 15 use X and Y. 7 use Y and Z but not X. 12 use X and Z. 50 use X and 41 use Y. 18 use Z only, and 26 use none. Draw a Venn diagram. A person is chosen at random. Find the probability that the person uses (a) Z, (b) exactly two of the services, (c) Y but not Z.

Answer .....

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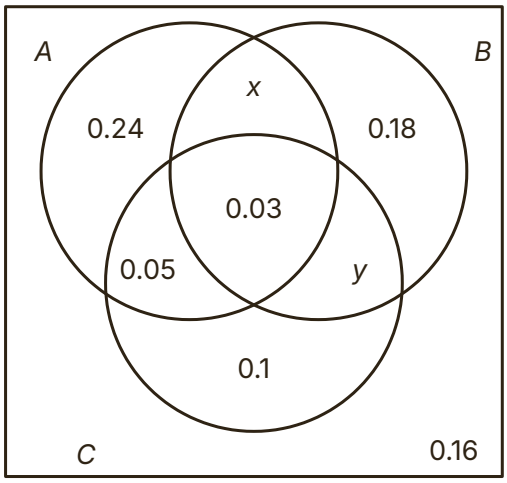
EXAMPLE 5

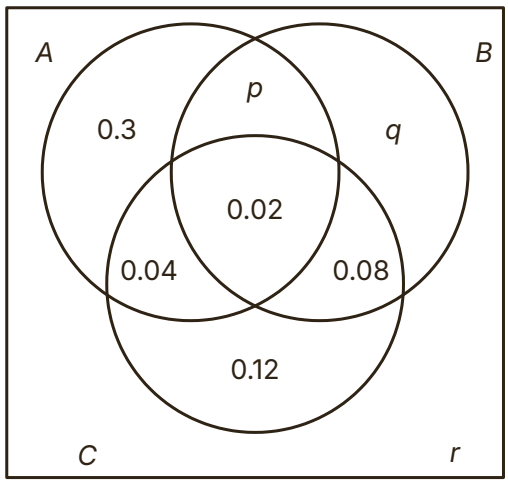
(i) $\mathrm{P}(A) = 0.55$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \text{ or } B) = 0.7$. Draw a Venn diagram of probabilities. (ii) The second Venn diagram shows the probabilities of three events. Given that $\mathrm{P}(A) = \mathrm{P}(C) + 0.1$, find x and y, and then $\mathrm{P}(B)$.



Hint (i) $\mathrm{P}(A) + \mathrm{P}(B)$ counts the overlap twice. (ii) Write $\mathrm{P}(A)$ and $\mathrm{P}(C)$ from the regions inside each circle, and use that all eight regions add to 1. That gives two equations in x and y.

B5 [7 marks]

The Venn diagram shows the probabilities of three events. Given that $\mathrm{P}(A) = 0.5$ and $\mathrm{P}(B) = 2\,\mathrm{P}(C)$, find p, q and r, and the probability that exactly one of the three events happens.



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Answer .....

C Practice

A-Level

C1 In a class of 30, 16 study history, 12 study geography and 5 study both. Mo draws a Venn diagram with 16 in "history only", 5 in the overlap and 12 in "geography only". He finds the number studying neither as $30 - 33 = -3$ and says the data must be wrong. Explain his mistake. Find the probability that a student chosen at random studies neither subject, and the probability that the student studies exactly one subject. [5 marks]

Answer .....

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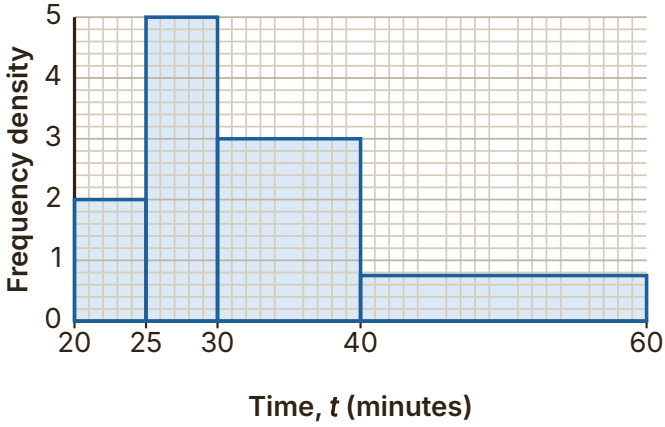
C2 The histogram shows the times, in minutes, taken by 80 runners to finish a race. Mia reads the heights of the bars as frequencies and writes $\mathrm{P}(\text{time} \geqslant 30) = \dfrac{3 + 0.75}{2 + 5 + 3 + 0.75} = 0.349$. Explain her mistake. Find $\mathrm{P}(\text{time} \geqslant 30)$, and estimate the probability that a runner took less than 35 minutes. [5 marks]



Answer .....

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C3 $\mathrm{P}(A) = 0.48$, $\mathrm{P}(B) = 0.35$ and $\mathrm{P}(A \text{ or } B) = 0.71$. Jade writes $\mathrm{P}(A \text{ or } B) = \mathrm{P}(A) + \mathrm{P}(B) = 0.83$ and says the question is wrong. Explain her mistake. Find $\mathrm{P}(A \text{ and } B)$ and the probability that neither event happens. [4 marks]

Answer .....

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C4 The table shows the journey times, in minutes, of 60 commuters. Leo estimates the number with a journey time under 30 minutes as $8 + 14 + 20 + \frac{5}{10} \times 12 = 48$. Explain his mistake. Estimate the probability that a commuter chosen at random has a journey time under 30 minutes, and the probability that the journey time is between 12 and 20 minutes. [5 marks]

Time, t (min)	Frequency
$0 \le t < 10$	8
$10 \le t < 15$	14
$15 \le t < 25$	20
$25 \le t < 40$	12
$40 \le t < 60$	6

Answer .....

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C5 $\mathrm{P}(A) = 0.4$, $\mathrm{P}(B) = 0.3$ and $\mathrm{P}(A \text{ and } B) = 0.1$. Amir says that "$A$ or B" means one of them but not both, so $\mathrm{P}(A \text{ or } B) = 0.5$. Explain his mistake. Find $\mathrm{P}(A \text{ or } B)$, the probability that exactly one event happens, and the probability that A and B do not both happen. [4 marks]

Answer .....

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D1 150 visitors to a museum were asked which of three galleries they visited: Art (A), History (H) and Science (S). 8 visited all three galleries. 23 visited Art and History, 20 visited History and Science, and 17 visited Art and Science. 62 visited Art, 58 visited History and 53 visited Science.

(a) Draw a Venn diagram to show this information. [4 marks]

Answer .....

(b) Find the probability that a visitor chosen at random visited exactly one gallery. [2 marks]

Answer .....

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(c) Find the probability that a visitor chosen at random visited Science and at least one other gallery. [2 marks]

Answer .....

(d) Find the probability that a visitor chosen at random visited at most one gallery. [3 marks]

Answer .....

Total for D1: 11 marks

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D2 A club has 90 members. 52 of them swim and 47 of them cycle.

(a) Find the least possible number and the greatest possible number of members who both swim and cycle. [4 marks]

Answer .....

(b) In fact 15 members do neither. Find the probability that a member chosen at random both swims and cycles. [3 marks]

Answer .....

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(c) Find the probability that a member chosen at random does exactly one of the two. [2 marks]

Answer .....

Total for D2: 9 marks

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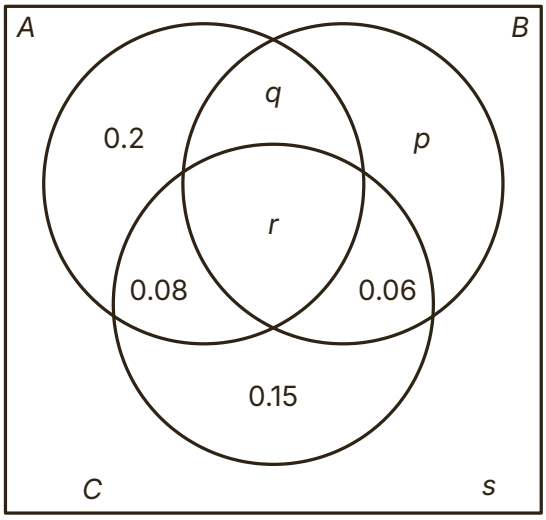
D3 The Venn diagram shows the probabilities of the events A, B and C. The probability that exactly one of the three events happens is 0.6, the probability that at least two of them happen is 0.23, and $\mathrm{P}(B \text{ and } C) = 0.1$. Find p, q, r and s, and find $\mathrm{P}(A)$. [9 marks]



Answer .....

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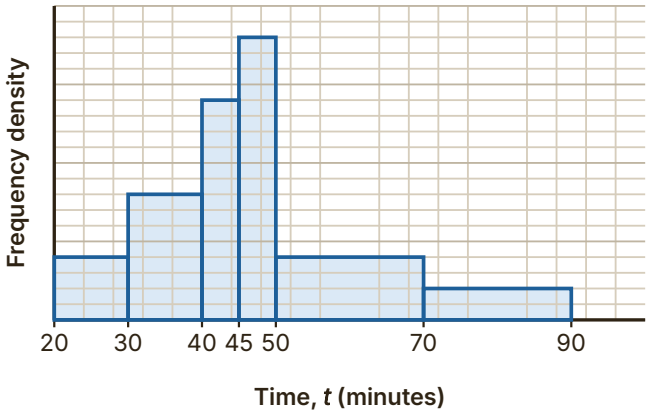
D4 The histogram shows the times taken by 160 runners to complete a fun run. No scale is shown on the frequency density axis. The bar for $30 \leqslant t < 40$ represents 32 runners.

(a) Find the probability that a runner chosen at random took less than 45 minutes. [3 marks]



Answer .....

(b) Estimate the probability that a runner chosen at random took between 42 and 60 minutes. [4 marks]

Answer .....

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(c) State the assumption you made in (b), and explain why your answer to (b) is an estimate. [2 marks]

Total for D4: 9 marks

D5

A survey of 200 people asked which of three newspapers P, Q and R they read. 7 read all three. 25 read P and Q, 19 read Q and R, and 22 read P and R. Zoe says: "The probability that a person reads exactly two newspapers is $\dfrac{25+19+22}{200} = 0.33$."

Zoe is wrong.

(a) Explain Zoe's mistake. [2 marks]

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(b) Find the correct probability that a person chosen at random reads exactly two newspapers. [3 marks]

Answer .....

(c) 88 people read P, 76 read Q, and 51 read none of the three. Find the probability that a person chosen at random reads R only. [4 marks]

Answer .....

Total for D5: 9 marks

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D6 The table shows the scores of the students in Year 12 and Year 13 who sat a test. A student is chosen at random.

Score, s	Year 12	Year 13
$0 \leq s < 20$	6	5
$20 \leq s < 40$	18	12
$40 \leq s < 60$	30	24
$60 \leq s < 80$	22	31
$80 \leq s < 100$	4	13

(a) Find the probability that the student is in Year 12 and scored less than 40. [2 marks]

Answer .....

(b) Find the probability that the student scored 60 or more. [2 marks]

Answer .....

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(c) The pass mark is 45. Estimate the probability that the student passed, and state one assumption you have made. [4 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Each of the 100 students in a year group studies at least one of French, German and Spanish. 50 study French, 45 study German and 40 study Spanish. 25 students study exactly two of the languages. 12 study French and German only, and 6 study German and Spanish only. A student is chosen at random. Find the probability that the student studies all three languages, the probability that the student studies French only, and the probability that the student studies Spanish but not French. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Let x students study all three. Adding the three totals counts a student with exactly one language once, with exactly two twice, and with all three three times.

Hint 2 So $50 + 45 + 40 = (\text{exactly one}) + 2(25) + 3x$, and $(\text{exactly one}) + 25 + x = 100$. Solve these for x.

Hint 3 With x known, the French and Spanish only region is $25 - 12 - 6$. Then each "only" region is a total minus the three regions inside it.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find probabilities from a sample space, a grouped frequency table and a histogram.	○ ○ ○
I can draw a Venn diagram for two or three events, working out from the centre.	○ ○ ○
I can find unknown probabilities in a Venn diagram from given conditions.	○ ○ ○
I can find the greatest and least possible size of an overlap.	○ ○ ○

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