Probability distributions

95 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-probability-distributions/

Date

A Do Now

Quick questions to start. Some use earlier work; some set a slip that the examiner reports name.

A1 Needed today [2 marks]

Two fair four-sided dice, each numbered 1 to 4, are rolled. Find the probability that the total score is 5.

Answer .....

A2 Needed today [3 marks]

A biased coin has $\mathrm{P}(\text{head}) = 0.3$. It is tossed three times. Find the probability of exactly one head.

Answer .....

A3 Needed today [2 marks]

Solve $\frac{k}{2} + \frac{k}{3} + \frac{k}{4} + \frac{k}{6} = 1$.

Answer .....

A4 Common slip [3 marks]

The random variable X takes the values $1, 2, 3, \ldots, 8$. List the values of X in each event: (i) $3 \leqslant X < 6$, (ii) "fewer than 4", (iii) "at most 2".

Answer .....

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PART 1 Finding an unknown constant

EXAMPLE 1 · I DO

The discrete random variable X has probability function $\mathrm{P}(X = x) = k(x + 2)$ for $x = 1, 2, 3, 4$, where k is a constant. Find k, write the distribution as a table, and find $\mathrm{P}(1 < X \leqslant 3)$ and $\mathrm{P}(X > 4)$.

Hint Substitute each value of x, add the four probabilities and set the total equal to 1. "$1 < X \leqslant 3$" means $X = 2$ or 3.

YOUR TURN · WE DO

B1 The discrete random variable X has probability function [5 marks]

$$\mathrm{P}(X = x) = \begin{cases} kx & x = 1, 2 \\ k(6 - x) & x = 3, 4 \end{cases}$$

where k is a constant.
Find k, $\mathrm{P}(X \geqslant 2)$ and $\mathrm{P}(2 \leqslant X < 4)$.

Answer .....

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EXAMPLE 2 · I DO

The discrete random variable X has the probability distribution shown, where a and b are constants. Given that $\mathrm{P}(X \geqslant 3) = 0.65$, find a and b.

x	1	2	3	4
$\mathrm{P}(X = x)$	a	b	0.25	$2a$

Hint Two unknowns need two equations. One comes from the given probability, the other from the fact that the probabilities add to 1.

YOUR TURN · WE DO

B2 The discrete random variable X has the probability distribution shown, where p and q are constants. Given that $\mathrm{P}(X \leqslant 0) = 0.4$, find p and q, and find $\mathrm{P}(-1 \leqslant X < 2)$. [5 marks]

x	-1	0	1	2	3
$\mathrm{P}(X = x)$	p	q	0.2	$2p$	0.1

Answer .....

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C1 The discrete random variable X has probability function $\mathrm{P}(X = x) = k(x-1)(7-x)$ for $x = 2, 3, 4, 5, 6$. Find k and $\mathrm{P}(X \leqslant 3)$. Explain, without further calculation, why $\mathrm{P}(X < 4) = \mathrm{P}(X > 4)$. [4 marks]

Answer .....

C2 Marta says that $\mathrm{P}(X = x) = \dfrac{1}{x^2}$ for $x = 1, 2, 3$ is a probability distribution. Show that she is wrong. The correct probability function is $\mathrm{P}(X = x) = \dfrac{k}{x^2}$ for $x = 1, 2, 3$. Find the exact value of k and find $\mathrm{P}(X \geqslant 2)$. [5 marks]

Answer .....

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PART 2 Distributions from outcomes

EXAMPLE 3 · I DO

Two fair spinners, each numbered 1, 2 and 3, are spun. The random variable X is the larger of the two scores (if the scores are equal, X is that score). Find the probability distribution of X as a table and as a probability mass function, and find $\mathrm{P}(X \geqslant 2)$.

Hint List the 9 equally likely outcomes in a grid. Count how many give each value of X. Look for a formula in x that fits the counts $1, 3, 5$.

YOUR TURN · WE DO

B3 Two fair four-sided dice, each numbered 1 to 4, are rolled. The random variable Y is the smaller of the two scores (if the scores are equal, Y is that score). Find the probability distribution of Y as a table, write it as a probability mass function, and find $\mathrm{P}(Y \geqslant 2)$. [5 marks]

Answer .....

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EXAMPLE 4 · I DO

Tom throws a dart at a target. Each throw hits with probability 0.4, independently. He throws until he hits the target or has thrown 3 times. Find the probability distribution of S, the number of throws, and find $\mathrm{P}(S \geqslant 2)$.

Hint $S = 1$: hit first time. $S = 2$: miss, then hit. $S = 3$ happens if the first two throws miss, whatever the third throw does.

YOUR TURN · WE DO

B4 A biased coin has $\mathrm{P}(\text{head}) = 0.2$. It is tossed until a head appears or it has been tossed 4 times. Find the probability distribution of T, the number of tosses, as a table, and find $\mathrm{P}(T > 2)$. [6 marks]

Answer .....

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C3 A spinner lands on red with probability 0.3. Leo spins it until it lands on red, or until he has spun it 4 times. S is the number of spins. Leo writes $\mathrm{P}(S = 4) = 0.7^3 \times 0.3 = 0.1029$. Explain his mistake, and find the probability distribution of S. [5 marks]

Answer .....

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C4 The word BANANA is written on 6 cards, one letter on each card. Two cards are taken at random, without replacement. X is the number of cards showing A. Nia says $X \sim \mathrm{B}(2, 0.5)$. Find the probability distribution of X, and explain why X does not have a binomial distribution. [6 marks]

Answer .....

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PART 3 Uniform distributions and two variables

EXAMPLE 5 · I DO

A bag holds 40 counters numbered 1 to 40. One counter is taken at random and X is its number. State a condition for X to have a discrete uniform distribution. Find $\mathrm{P}(X = 7)$, the probability that X is a multiple of 3, and $\mathrm{P}(12 < X \leqslant 30)$. Find the value of c for which $\mathrm{P}(X \leqslant c) = 0.3$.

Hint In a discrete uniform distribution on n values each value has probability $\frac{1}{n}$, so every probability is (number of values in the event) $\div n$. Count the values carefully at each end.

YOUR TURN · WE DO

B5 The random variable X has a discrete uniform distribution over the integers $11, 12, \ldots, 35$. Find $\mathrm{P}(X = 20)$, the probability that X is even, and $\mathrm{P}(15 \leqslant X < 28)$. Given that $\mathrm{P}(X \leqslant c) = 0.4$, find c. [5 marks]

Answer .....

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C5 The random variable X has a discrete uniform distribution over the integers $5, 6, \ldots, 24$. Find $\mathrm{P}(X = 10)$, the probability that X is odd, and $\mathrm{P}(8 < X \leqslant 17)$. Given that $\mathrm{P}(X \leqslant c) = 0.35$, find c. [5 marks]

Answer .....

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C6 The random variable X has the distribution shown. The random variable Y, independent of X, has $\mathrm{P}(Y = 1) = 0.7$ and $\mathrm{P}(Y = 2) = 0.3$. Sam writes $\mathrm{P}(X + Y = 3) = \mathrm{P}(X = 1) \times \mathrm{P}(Y = 2) = 0.15$. Explain his mistake. Find $\mathrm{P}(X + Y = 3)$ and $\mathrm{P}(X > Y)$. [6 marks]

x	**1**	**2**	**3**
$\mathrm{P}(X = x)$	0.5	0.3	0.2

Answer .....

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D1 A fair six-sided dice, numbered 1 to 6, and a fair four-sided dice, numbered 1 to 4, are rolled. The random variable D is the difference between the two scores (the larger minus the smaller, or 0 if they are equal).

(a) Find the probability distribution of D. [4 marks]

Answer .....

(b) Find $\mathrm{P}(D \geqslant 2)$. [2 marks]

Answer .....

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(c) In a game a player wins if $D \geqslant 3$. Find the expected number of wins when the game is played 48 times. [2 marks]

Answer .....

(d) Find the probability that a player wins exactly one of two games. [2 marks]

Answer .....

Total for D1: 10 marks

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D2

> Ama says: "The probabilities add to 1, so $k(0+1+4+9)=1$ and $k=\dfrac{1}{14}$."

The discrete random variable X has probability function $\mathrm{P}(X=x)=k(x^2+1)$ for $x=0,1,2,3$.

(a) Explain Ama's mistake. Find the correct value of k. [3 marks]

Answer .....

(b) Write the probability distribution of X as a table. [2 marks]

Answer .....

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(c) Two independent observations X_1 and X_2 are taken. Find $\mathrm{P}(X_1 + X_2 = 3)$. [4 marks]

Answer .....

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The random variable X has a discrete uniform distribution over the integers $1, 2, \ldots, n$, where $n \geqslant 4$. The random variable Y, independent of X, has probability function $\mathrm{P}(Y = y) = \dfrac{y}{10}$ for $y = 1, 2, 3, 4$. Given that $\mathrm{P}(X > Y) = 0.7$, find n. For this value of n, find $\mathrm{P}(X = Y)$ and $\mathrm{P}(X < Y)$. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Fix the value of Y first. If $Y = y$, how many of the n values of X are bigger than y?

Hint 2 Add $\mathrm{P}(Y = y) \times \mathrm{P}(X > y)$ over $y = 1, 2, 3, 4$. Simplify before you set it equal to 0.7.

Hint 3 You should reach $\dfrac{n-3}{n}$. For $\mathrm{P}(X = Y)$, each y contributes $\mathrm{P}(Y = y) \times \dfrac{1}{n}$. The three cases $X > Y$, $X = Y$, $X < Y$ cover everything.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the probability distribution of a discrete random variable as a table and as a probability mass function.	○ ○ ○
I can find an unknown constant or unknown probabilities using the fact that the probabilities add up to 1.	○ ○ ○
I can use the discrete uniform distribution and find probabilities of events such as $a < X \leqslant b$.	○ ○ ○
I can find probabilities that involve two independent random variables.	○ ○ ○

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