Tree diagrams

116 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-tree-diagrams/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Needed today [2 marks]

A and B are independent, with $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.6$. Find the probability that both happen and the probability that neither happens.

Answer .....

A2 Needed today [2 marks]

A bag holds 5 red and 4 blue counters. A counter is taken, its colour noted, and it is put back. A second counter is then taken. Find the probability that the two counters are different colours.

Answer .....

A3 Common slip [3 marks]

A bag holds 6 red and 4 green counters. Two counters are taken without replacement. Sam writes $\mathrm{P}(\text{both red}) = \frac{6}{10} \times \frac{6}{10} = 0.36$. Find the correct value, and the probability that both are green.

Answer .....

A4 Needed today [2 marks]

A fair six-sided die is thrown three times. Find the probability of at least one six.

Answer .....

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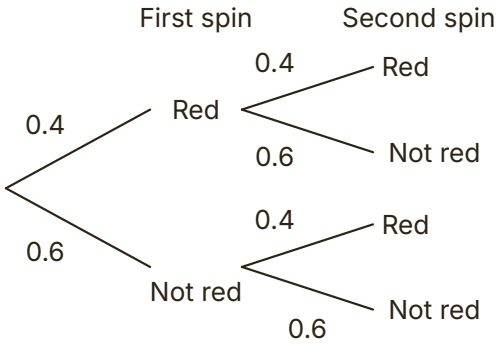
B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A spinner lands on red with probability 0.4. It is spun twice. Draw a tree diagram, and find the probability of (a) two reds, (b) exactly one red, (c) at least one red.



Hint The second spin does not depend on the first, so the second-stage branches repeat 0.4 and 0.6. Multiply along each path, then add the paths that give the event.

B1 A biased coin lands heads with probability 0.65. It is thrown twice. Find the probability of (a) two heads, (b) exactly one head, (c) at least one tail. [4 marks]

Answer .....

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EXAMPLE 2

A bag holds 5 red and 3 blue counters. Two counters are taken without replacement. Find the probability that (a) they are the same colour, (b) at least one is blue.

Hint After a red is taken there are 4 red out of 7. Both the top and the bottom of the second fraction change.

B2 A box holds 7 milk chocolates and 5 dark chocolates. Two are taken at random without replacement. Find the probability that (a) both are dark, (b) one of each kind is taken, (c) the second chocolate taken is milk. [5 marks]

Answer .....

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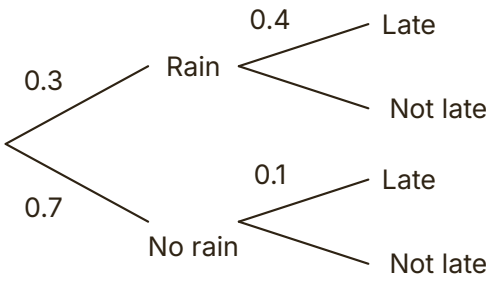
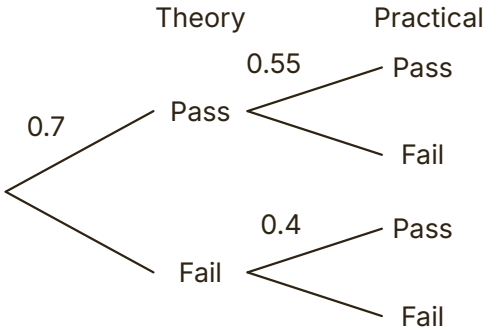
EXAMPLE 3

On a school day the probability that it rains is 0.3. If it rains, the probability that Leo is late is 0.4. If it does not rain, the probability that he is late is 0.1. Find the probability that Leo is late, and the probability that it rains and he is not late. Are the events "it rains" and "Leo is late" independent?



Hint Fill in the missing branches first: each pair from one point adds to 1. "Late" is at the end of two paths.

B3 The probability that Ama passes her theory test at the first attempt is 0.7. If she passes it, the probability that she passes her practical test at the first attempt is 0.55. If she fails the theory test, the probability that she passes the practical at the first attempt is 0.4. Find the probability that she (a) passes the practical at the first attempt, (b) passes the theory but fails the practical, (c) passes exactly one of the two tests at the first attempt. [5 marks]



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Answer .....

EXAMPLE 4

A bag holds 4 red, 3 green and 2 yellow counters. Three counters are taken without replacement. Find the probability that (a) the three counters are all different colours, (b) the three counters are all the same colour.

Hint Work out one order, such as red then green then yellow, then count how many orders give the same event. All the orders have the same probability.

B4 An archer hits the target with probability 0.7 on each shot, independently. She takes three shots. Find the probability that she hits the target (a) exactly twice, (b) at least twice, (c) at most twice. [5 marks]

Answer .....

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EXAMPLE 5

A biased coin lands heads with probability p, where $p > 0.5$. It is thrown twice. The probability of exactly one head is 0.42. Find p.

Hint Exactly one head has two paths: $p(1-p)$ each. Set $2p(1-p) = 0.42$ and solve the quadratic. Then use $p > 0.5$.

B5 [6 marks]

A bag holds n red counters and 3 blue counters. Two counters are taken without replacement. The probability that both are red is $\frac{7}{15}$. Show that $4n^2 - 25n - 21 = 0$, and find n.

Answer .....

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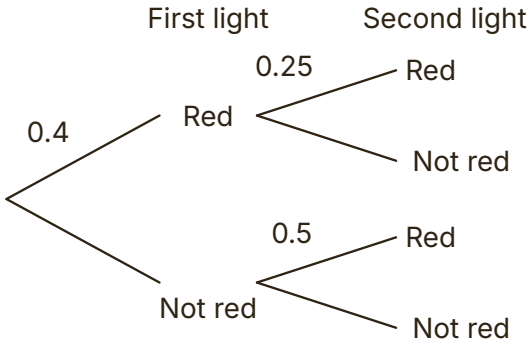
C1 At the first set of traffic lights the probability of a red light is 0.4. If the first light is red, the probability that the second is red is 0.25. If the first is not red, the probability that the second is red is 0.5. Dan finds the probability that both lights are red as $0.4 + 0.25 = 0.65$. Explain his mistake. Find the probability that both lights are red, and the probability that exactly one light is red. [4 marks]



Answer .....

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C2 A bag holds 3 red and 7 white counters. Two counters are taken without replacement. Ella writes $\mathrm{P}(\text{two white}) = 0.7 \times 0.7 = 0.49$. Explain her mistake. Find the correct probability of two white counters, and the probability of at least one red counter. [4 marks]

Answer .....

C3 A fair six-sided die is thrown four times. Leo says the probability of at least one six is $4 \times \frac{1}{6} = \frac{2}{3}$. Explain why his method is wrong, and find the correct probability to 3 significant figures. [4 marks]

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C4 A bag holds 3 red, 4 blue and 5 green counters. Three counters are taken without replacement. Kim writes $\mathrm{P}(\text{one of each colour}) = \frac{3}{12} \times \frac{4}{11} \times \frac{5}{10} = \frac{1}{22}$. Explain her mistake, and find the correct probability. [4 marks]

Answer .....

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C5 A bag holds 4 red and 6 blue counters. Two counters are taken without replacement. Jo says the probability that the second counter is red is $\frac{3}{9}$, "because one red counter has gone". Explain why Jo is wrong, and find the probability that the second counter is red. [4 marks]

Answer .....

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D1 A box holds 4 red and 6 blue counters. Counters are taken from the box at random, one at a time and without replacement, until a red counter is taken or until three counters have been taken.

(a) Draw a tree diagram to show this, with the probability on each branch. [2 marks]

Answer .....

(b) Find the probability that exactly two counters are taken. [2 marks]

Answer .....

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(c) Find the probability that no red counter is taken. [3 marks]

Answer .....

(d) Find the probability that exactly three counters are taken. [3 marks]

Answer .....

Total for D1: 10 marks

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D2 A bag holds n green counters and 4 yellow counters. Two counters are taken at random without replacement. The probability that the two counters are different colours is $\frac{8}{15}$.

(a) Show that $n^2 - 8n + 12 = 0$. [4 marks]

Answer .....

(b) Find the possible values of n. [2 marks]

Answer .....

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(c) There are more green counters than yellow. Find the probability that both counters taken are green. [3 marks]

Answer .....

Total for D2: 9 marks

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D3 Ali and Beth take turns to throw a fair six-sided die, starting with Ali. The first person to throw a six wins. Each person has at most two throws, so there are at most four throws; if no six is thrown, the game is a draw. Find the probability that Beth wins, and the probability that the game is a draw. [7 marks]

Answer .....

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D4 Each morning Kirsty takes the bus with probability 0.6; otherwise she cycles. If she takes the bus, the probability that she is late is 0.15. If she cycles, the probability that she is late is 0.05.

(a) Draw a tree diagram to show this information. [2 marks]

Answer .....

(b) Find the probability that Kirsty is late on a given morning. [2 marks]

Answer .....

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(c) Show that the events "takes the bus" and "is late" are not independent, and explain what this means in context. [3 marks]

(d) Mornings are independent of each other. Find the probability that Kirsty is late on exactly one of two mornings. [3 marks]

Answer .....

Total for D4: 10 marks

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D5

A bag holds 5 red and 5 blue counters. Two counters are taken without replacement. Mia says: "There are as many red as blue, so getting two the same colour and getting two different colours are equally likely. Each has probability 0.5."

Mia is wrong.

(a) Show that Mia is wrong. [3 marks]

Answer .....

(b) A third counter is now taken from the bag. Find the probability that all three counters are the same colour. [3 marks]

Answer .....

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(c) Find the probability that exactly two of the three counters are red. [3 marks]

Answer .....

Total for D5: 9 marks

D6 Each item made by a machine is faulty with probability p, independently of other items. When two items are checked, the probability that at least one is faulty is 0.19.

(a) Find p. [3 marks]

Answer .....

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(b) Three items are checked. Find the probability that exactly one is faulty. [3 marks]

Answer .....

(c) Items are checked one at a time until a faulty item is found, or until three items have been checked. Find the probability that three items are checked. [2 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A bag holds 10 counters. Some are red and the rest are blue, and there are more red counters than blue. Two counters are taken at random without replacement, and the probability that they are the same colour is $\frac{7}{15}$. Three counters are then taken from the full bag of 10, without replacement. Find the probability that more of these three counters are red than blue. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Let there be r red counters, so $10 - r$ blue. The two same-colour paths give $\dfrac{r(r-1) + (10-r)(9-r)}{90} = \dfrac{7}{15}$.

Hint 2 This gives $r^2 - 10r + 24 = 0$. Choose the root that gives more red than blue.

Hint 3 More red than blue in three counters means exactly two red or three red. Exactly two red has three orders.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can draw a tree diagram and multiply along branches, then add branches.	○ ○ ○
I can change both the top and the bottom of the fractions when counters are not replaced.	○ ○ ○
I can use 1 minus the probability of none for at least one.	○ ○ ○
I can form and solve an equation for an unknown probability or number of counters.	○ ○ ○

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