Two-tailed tests

98 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-two-tailed-tests/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that the examiner reports say costs marks.

A1 Common slip [2 marks]

$X \sim \mathrm{B}(16, 0.3)$. Find $\mathrm{P}(X \geqslant 8)$.

Answer .....

A2 Last lesson [2 marks]

These are the six steps of a hypothesis test, out of order: Compare; Model; Conclude; Hypotheses; Calculate assuming H_0; Level and tails. Write them in the correct order.

Answer .....

A3 Common slip [2 marks]

A test is about the proportion p of seeds that grow, which is claimed to be 0.3. Jo writes $\mathrm{H}_0 : X = 0.3$, $\mathrm{H}_1 : X > 0.3$. Say what is wrong and write the hypotheses correctly.

Answer .....

A4 Common slip [2 marks]

In a one-tailed test at the 5% level, 9 successes are observed and, assuming H_0, $\mathrm{P}(X \geqslant 9) = 0.031$. Sam writes "$9 > 0.05$, so do not reject H_0". Correct his comparison and his decision.

Answer .....

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A5 Last lesson [3 marks]

$X \sim \mathrm{B}(18, 0.25)$. Find the critical region for a test of $\mathrm{H}_0 : p = 0.25$ against $\mathrm{H}_1 : p > 0.25$ at the 5% level.

Answer .....

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PART 1 Two-tailed tests with a p-value

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EXAMPLE 1 · I DO

Over a long period, 40% of the visitors to a museum have bought a guidebook. The museum changes its guidebook. A random sample of 25 visitors is taken after the change, and 5 of them buy the new guidebook. Test, at the 5% level of significance, whether the proportion of visitors who buy a guidebook has changed.

Hint "Changed" has no direction, so the test is two-tailed. Use the tail on the side of the observed value, and compare with half of 5%.

YOUR TURN · WE DO

B1 A survey found that 35% of the households in a town have a pet. A council officer thinks that this proportion has changed. She takes a random sample of 20 households and finds that 12 of them have a pet. She tests, at the 5% level, whether the proportion has changed. Do the test one step at a time.

(a) Step 1 Model: define X and state its distribution if H_0 is true. [1 mark]

Answer .....

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(b) Step 2 Hypotheses: write H_0 and H_1, and give the word that makes the test two-tailed. [1 mark]

Answer .....

(c) Step 3 Level and tails: state the greatest probability allowed in each tail. [1 mark]

Answer .....

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(d) Step 4 Calculate assuming H_0: decide which tail to use and find the probability. [2 marks]

Answer .....

(e) Step 5 Compare. [1 mark]

Answer .....

(f) Step 6 Conclude, in context. [1 mark]

Answer .....

Answer .....

Total for B1: 7 marks

PRACTICE · YOU DO

C1 Records show that 15% of patients miss their appointment at a surgery. The surgery starts sending text reminders. In a random sample of 40 appointments after the change, 11 are missed. Kai tests, at the 5% level, whether the proportion of missed appointments has changed. He writes: "$\mathrm{H}_0 : p = 0.15$, $\mathrm{H}_1 : p \neq 0.15$. $\mathrm{P}(X \geqslant 11) = 0.02992$. $0.02992 < 0.05$, so reject H_0." Explain Kai's mistake and give the correct conclusion. [4 marks]

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C2 A computer program should output a 1 with probability 0.5 each time it runs. In 30 runs it outputs 21 ones. Test, at the 1% level of significance, whether the probability of a 1 is different from 0.5. Do the test one step at a time.

(a) Steps 1 and 2: the model and the hypotheses. [2 marks]

Answer .....

(b) Step 3: the probability allowed in each tail. [1 mark]

Answer .....

(c) Step 4: the probability, assuming H_0. [2 marks]

Answer .....

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(d) Steps 5 and 6: compare, and conclude in context. [2 marks]

Answer .....

Total for C2: 7 marks

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PART 2 Two-tailed critical regions

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EXAMPLE 2 · I DO

A seed company states that 25% of its seeds produce a white flower. A gardener will grow 40 of the seeds to test whether the proportion is different from 25%.

(a) Find the critical region for a test at the 5% level of significance. The probability in each tail should be at most 2.5%.

(b) State the actual significance level of the test.

(c) 15 of the 40 seeds produce a white flower. Comment on this in light of the critical region.

Hint Find each tail on its own. Write the probability on each side of the boundary, so the reader sees why the boundary is there.

YOUR TURN · WE DO

B2 A firm says that 45% of its workers cycle to work. A manager will ask a random sample of 30 workers, to test whether the proportion this year is different.

(a) State the hypotheses, and say why the test is two-tailed. [2 marks]

Answer .....

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(b) Find the critical region at the 5% level, with the probability in each tail at most 2.5%. [4 marks]

Answer .....

(c) State the actual significance level. [1 mark]

Answer .....

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EXAMPLE 3 · I DO

Records show that 8% of the parcels from a depot are damaged. The depot starts a new packing method. A random sample of 25 parcels is checked.

(a) Find the critical region for a test, at the 5% level, of whether the proportion of damaged parcels has changed. The probability in each tail should be at most 2.5%.

(b) Explain why the critical region has only one part.

(c) 6 of the 25 parcels are damaged. Complete the test.

Hint Start the lower tail at $X = 0$. If even $\mathrm{P}(X = 0)$ is more than 0.025, there is no lower tail. Compare 6 with np, not with how small 6 looks.

YOUR TURN · WE DO

B3 A delivery firm says that 10% of its orders arrive late. A shop owner checks a random sample of 12 orders to test whether the proportion of late orders is different from 10%, at the 5% level, with the probability in each tail at most 2.5%.

(a) Find the critical region, explain why it has only one part, and state its actual significance level. [5 marks]

Answer .....

Total for B2: 8 marks

(d) 19 of the 30 workers cycle to work. Comment on this in light of the critical region. [1 mark]

Answer .....

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(b) 4 of the 12 orders arrive late. Complete the test. [2 marks]

PRACTICE · YOU DO

C3 $X \sim \mathrm{B}(24, p)$ is used to test $\mathrm{H}_0 : p = 0.3$ against $\mathrm{H}_1 : p \neq 0.3$ at the 5% level, with each tail at most 2.5%. Mia writes: "The critical region is $\mathrm{P}(X \leqslant 2) = 0.01187$ and $\mathrm{P}(X \geqslant 13) = 0.01150$, so the critical region is $2 < X < 13$." Explain her two mistakes. Write the correct critical region and its actual significance level, and say what happens if $X = 13$ is observed. [5 marks]

Answer .....

Total for C3: 7 marks

Answer .....

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C4 A post office says that 5% of its letters are delivered late. A manager checks a random sample of 40 letters. Find the critical region at the 5% level, with the probability in each tail at most 2.5%. Explain why it has no lower part. [5 marks]

Answer .....

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PART 3 The level and the choice of tails

EXAMPLE 4 · I DO

A machine fills packets, and 15% of the packets are underweight. After the machine is serviced, a random sample of 20 packets contains no underweight packets.

(a) Test, at the 10% level of significance, whether the proportion of underweight packets has changed.

(b) Would the conclusion be the same at the 1% level? Explain.

Hint At 10% each tail is 5%; at 1% each tail is 0.5%. The probability you calculate is the same; only the level it is compared with changes.

YOUR TURN · WE DO

B4 A football coach says that 70% of the penalties taken by his players are scored. After a new training method, the players take 30 penalties and score 25. Test, at the 10% level of significance, whether the proportion of penalties scored has changed. [7 marks]

Answer .....

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C5 The old glue used by a furniture maker lets 14% of joints fail. A new glue is said to make fewer of the joints fail. In a random sample of 45 joints made with the new glue, 2 fail. Zoe tests the claim at the 5% level. She writes: "$\mathrm{H}_0 : p = 0.14$, $\mathrm{H}_1 : p \neq 0.14$. $\mathrm{P}(X \leqslant 2) = 0.03900$. $0.03900 > 0.025$, so do not reject H_0." Explain Zoe's mistake and give the correct conclusion. [4 marks]

Answer .....

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C6 Over a long period, 25% of the customers at a café buy a cake. The café changes its menu. Of a random sample of 16 customers after the change, 8 buy a cake. Test, at the 10% level of significance, whether the proportion of customers who buy a cake has changed. [6 marks]

Answer .....

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D1

A machine produces items, of which 12% are faulty. After a repair, the manager wants to know whether the proportion of faulty items has changed. In a random sample of 30 items, none is faulty. Priya writes: "$\mathrm{H}_0 : p = 0.12$, $\mathrm{H}_1 : p < 0.12$. $\mathrm{P}(X = 0) = 0.02160$. $0.02160 < 0.05$, so accept H_1. This proves that the repair has reduced the proportion of faulty items."

Priya's working has errors.

(a) Describe three errors in Priya's working. [3 marks]

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(b) Carry out the test correctly, at the 5% level. [6 marks]

Answer .....

(c) Would the conclusion be different if the test were carried out at the 4% level? Explain. [2 marks]

Total for D1: 11 marks

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D2 In a large wood, 45% of the beetles of one species are red. A biologist finds a random sample of 50 beetles of this species at a new site, and 32 of them are red. Test, at the 1% level of significance, whether the proportion of red beetles at the new site is different from the proportion in the wood. State your hypotheses clearly. [7 marks]

Answer .....

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No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 To check whether a coin is biased, Lena tosses it 10 times and uses this rule. With 0, 1, 9 or 10 heads she decides that the coin is biased. With 3 to 7 heads she decides that it is not biased. With 2 or 8 heads she tosses the coin 10 more times and decides that it is biased if the total number of heads in all 20 tosses is 5 or fewer, or 15 or more. Find the probability that Lena decides that a fair coin is biased. Compare this with the actual significance level of a single two-tailed test on 20 tosses with critical region $X \leqslant 5$ or $X \geqslant 15$. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 For a fair coin the number of heads in 10 tosses is $\mathrm{B}(10, 0.5)$. Split the ways to decide "biased" by the result of the first 10 tosses.

Hint 2 After 2 heads in the first 10, a total of 5 or fewer needs 3 or fewer heads in the next 10; a total of 15 or more is impossible. After 8 heads, use the same idea the other way.

Hint 3 Add: $\mathrm{P}(\text{first 10 decide}) + 2 \times \mathrm{P}(2 \text{ heads}) \times \mathrm{P}(Y \leqslant 3)$, by symmetry.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can tell from the wording when a test is two-tailed and write H1 with the not-equal sign.	○ ○ ○
I can carry out a two-tailed test by comparing the probability in the correct tail with half the level.	○ ○ ○
I can find a two-tailed critical region, with each tail at most half the level, and its actual significance level.	○ ○ ○
I can write a conclusion in context that says whether there is evidence of a change.	○ ○ ○

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