Variance and standard deviation

112 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-variance-and-standard-deviation/

Date

A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that the examiner reports say costs marks.

A1 Needed today [2 marks]

For the values $4, 7, 8, 10, 11$ find $\sum x$, the mean and $\sum x^2$.

Answer .....

A2 Common slip [2 marks]

A frequency table has values $x = 1, 2, 3$ with frequencies $2, 3, 5$. Find $\sum fx$ and $\sum fx^2$.

Answer .....

A3 Common slip [2 marks]

A student finds that the variance of a data set is 20.25 and writes "standard deviation $= 20.25$". Find the standard deviation.

Answer .....

A4 Needed today [2 marks]

The values $x = 0, 1, 2$ have frequencies $3, 5, 2$. Find the mean.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the variance and the standard deviation of $3, 5, 6, 9, 12$, using both forms of the formula.

Hint First form: $\sigma^2 = \dfrac{\sum(x - \bar{x})^2}{n}$.

Second form: $\sigma^2 = \dfrac{\sum x^2}{n} - \bar{x}^2$, "the mean of the squares minus the square of the mean".

B1 The numbers of goals scored by a team in six matches are $12, 15, 9, 18, 11, 13$. Find the mean, the variance and the standard deviation. [4 marks]

Answer .....

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EXAMPLE 2

For 20 values, $\sum x = 364$ and $\sum x^2 = 7028$. Find the mean, S_{xx} and the standard deviation.

Hint $S_{xx} = \sum x^2 - \dfrac{(\sum x)^2}{n}$, and $\sigma = \sqrt{\dfrac{S_{xx}}{n}}$.

B2 The lengths, x cm, of 15 fish are summarised by $\sum x = 252$ and $\sum x^2 = 4578$.

(a) Find the mean and S_{xx}. [2 marks]

Answer .....

(b) Find the standard deviation. [2 marks]

Answer .....

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(c) A 16th fish of length 16.8 cm is added. Without further calculation, state the effect on the mean and on the standard deviation, giving a reason. [2 marks]

EXAMPLE 3

The numbers of eggs, x, in 40 nests are shown in the table. Find the mean and the standard deviation.

Eggs, x	**1**	**2**	**3**	**4**	**5**
Frequency, f	4	9	12	10	5

Hint Add rows for fx and fx^2. Remember fx^2 means $f \times x^2$, not $(fx)^2$.

B3 The numbers of absences, x, of 50 pupils in a term are shown in the table. Find the mean and the standard deviation. [5 marks]

Absences, x	**0**	**1**	**2**	**3**	**4**
Frequency, f	6	11	15	10	8

Answer .....

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EXAMPLE 4

The times, t minutes, of 30 journeys are shown in the table. Estimate the mean and the standard deviation.

Time, t (min)	Frequency
$0 \leq t < 10$	5
$10 \leq t < 20$	12
$20 \leq t < 30$	8
$30 \leq t < 50$	5

Hint Use the midpoint of each class as its value. The answers are estimates, as the actual times are not known.

B4 The masses, m kg, of 40 bags of sand are shown in the table.

Mass, m (kg)	Frequency
$0 \leq m < 4$	7
$4 \leq m < 8$	15
$8 \leq m < 12$	10
$12 \leq m < 20$	8

(a) Estimate the mean mass. [2 marks]

Answer .....

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(b) Estimate the standard deviation. [4 marks]

C Practice

A-Level

C1 The table shows the numbers of rooms, x, booked by 20 groups. Priya squares each fx and finds $\sum(fx)^2 = 1453$, then $\sigma^2 = \frac{1453}{20} - 3.55^2 = 60.0$. Explain her mistake and find the standard deviation correctly. [5 marks]

Rooms, x	2	3	4	5
Frequency, f	3	7	6	4

Answer .....

Total for B4: 6 marks

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C2 For 12 values, $\sum x = 150$ and $\sum x^2 = 1956$. Leo says "the standard deviation is 6.75". Explain his mistake and find the standard deviation. [4 marks]

C3 For 8 values, $\sum x = 122.3$ and $\sum x^2 = 1878.6$. Mo rounds the mean to 15.3 and finds $\sigma = \sqrt{234.825 - 15.3^2} = 0.857$. Explain why his answer is inaccurate and find σ correctly. [4 marks]

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C4 A data set of 10 values has mean 20 and standard deviation 4. A value of 20 is added. Sara says "the mean and the standard deviation are both unchanged, because the new value equals the mean". Explain why Sara is only partly right, and find the new standard deviation. [4 marks]

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C5 Here are 12 values: [5 marks]

$$2,\ 5,\ 6,\ 7,\ 7,\ 8,\ 8,\ 9,\ 10,\ 11,\ 12,\ 23$$

Find the mean and the standard deviation, and state how many values are more than one standard deviation from the mean.

Answer .....

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D Exam-style questions

Edexcel exam style

D1 The daily mean temperatures, $t\ ^\circ\mathrm{C}$, at a weather station on the 31 days of one month give $\sum t = 436.4$ and $\sum t^2 = 7038.92$. (These summary values are for this question only; they are not taken from the large data set.)

(a) Find the mean and the standard deviation of the temperatures. [4 marks]

Answer .....

(b) A value of 14.2 was recorded wrongly as 41.2. Find the correct mean and standard deviation. [4 marks]

Answer .....

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(c) At a second station in the same month the mean is $15.6\ ^\circ\mathrm{C}$ and the standard deviation is $1.8\ ^\circ\mathrm{C}$. Use your answers to (b) to compare the temperatures. [2 marks]

Total for D1: 10 marks

D2

Class A has 12 students, whose test marks have mean 64 and standard deviation 8. Class B has 18 students, whose marks have mean 70 and standard deviation 6. Tom says: "For all 30 students the mean is $\frac{64+70}{2} = 67$ and the standard deviation is $\frac{8+6}{2} = 7$."

Tom is wrong.

(a) Find $\sum x$ and $\sum x^2$ for each class. [4 marks]

Answer .....

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(b) Find the mean and the standard deviation of the marks of all 30 students, and say why Tom's method is wrong. [5 marks]

Total for D2: 9 marks

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D3 Five values are $4, 7, 9, a$ and b, where $a < b$. The mean of the five values is 7 and the variance is 5.2. Find a and b. [7 marks]

Answer .....

D4 The masses, m kg, of 70 adults are shown in the table.

(a) Estimate the mean mass. [3 marks]

Mass, m (kg)	Frequency
$50 \leq m < 60$	8
$60 \leq m < 70$	20
$70 \leq m < 80$	25
$80 \leq m < 90$	12
$90 \leq m < 110$	5

Answer .....

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(b) Estimate the standard deviation. [4 marks]

Answer .....

(c) Estimate how many of the adults have a mass within one standard deviation of the mean. [3 marks]

Answer .....

Total for D4: 10 marks

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D5 A data set of 20 values has mean 30 and standard deviation 5.

(a) Find $\sum x$ and $\sum x^2$. [3 marks]

Answer .....

(b) The values 22 and 38 are added. Find the new mean and standard deviation. [4 marks]

Answer .....

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(c) Explain, without calculating, why the standard deviation went up. [2 marks]

Total for D5: 9 marks

D6 The numbers of times, x, that a group of people visited a cinema last year are shown in the table. The mean number of visits is 3.

Visits, x	**1**	**2**	**3**	**4**	**5**
Frequency, f	5	p	12	8	3

(a) Find the value of p. [3 marks]

Answer .....

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(b) Find the standard deviation. [3 marks]

Answer .....

(c) Is a person who visited 5 times more than 1.5 standard deviations above the mean? Show your working. [2 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A data set of 10 values has mean 12 and standard deviation 3. Two more values, x and y with $x < y$, are added. The mean of the 12 values is still 12 and the standard deviation is still 3. Find x and y. [8 marks]

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 The mean is unchanged, so $x + y = 2 \times 12$.

Hint 2 Work with $\sum(v - \bar{v})^2$: for the ten values it is 10×3^2. What must it be for the twelve values?

Hint 3 The two new values must add 2×3^2 to $\sum(v - 12)^2$, and they are the same distance from 12.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the variance and standard deviation from raw data, and from summary statistics.	○ ○ ○
I can find the standard deviation of a frequency table and estimate it for grouped data.	○ ○ ○
I can combine two groups, correct a wrong value and find missing values from the mean and variance.	○ ○ ○
I can say how adding values changes the standard deviation, and compare distributions in context.	○ ○ ○

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