Approximating a binomial distribution

122 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-approximating-a-binomial-distribution/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that the examiner reports say costs marks.

A1 Needed today [2 marks]

$Y \sim \mathrm{N}(40, 5^2)$. Find $\mathrm{P}(Y > 47.5)$.

Answer .....

A2 Common slip [2 marks]

$Y \sim \mathrm{N}(120, 64)$. Kai types mean 120 and standard deviation 64 into his calculator and gets $\mathrm{P}(Y < 110) = 0.4379$. Correct his answer.

Answer .....

A3 Needed today [2 marks]

$X \sim \mathrm{B}(25, 0.4)$. Find $\mathrm{P}(X > 12)$.

Answer .....

A4 Needed today [2 marks]

$X \sim \mathrm{B}(20, 0.5)$ is discrete and $Y \sim \mathrm{N}(10, 5)$ is continuous. Find $\mathrm{P}(X = 10)$, and state $\mathrm{P}(Y = 10)$.

Answer .....

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B Example, then your turn

A-Level

Every test has the same six steps. Step 1 Model: say what is measured or counted, and its distribution if H_0 is true. Step 2 Hypotheses: H_0 and H_1 in the parameter p, never in the sample value. Step 3 Level and tails: the significance level, and one tail or two. Step 4 Calculate assuming H_0: the probability of the observed value or a more extreme one, here from the normal approximation with a continuity correction. Step 5 Compare: that probability with the level, or the observed value with the critical region. Step 6 Conclude: reject or do not reject H_0, then one sentence in context. The examples list how the continuity correction moves each boundary.

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EXAMPLE 1

For each binomial distribution, say whether a normal approximation is suitable. If it is, state the approximating distribution.
(i) $X \sim \mathrm{B}(160, 0.45)$ (ii) $X \sim \mathrm{B}(300, 0.01)$ (iii) $X \sim \mathrm{B}(8, 0.5)$

Hint Two conditions: n is large, and p is close to 0.5. A quick check is $np > 5$ and $n(1-p) > 5$. The mean is np and the variance is $np(1-p)$.

B1 For each distribution, say whether a normal approximation is suitable, giving both conditions. If it is, state the approximating distribution.

(a) $X \sim \mathrm{B}(250, 0.56)$ [2 marks]

Answer .....

(b) $X \sim \mathrm{B}(250, 0.012)$ [2 marks]

Total for B1: 4 marks

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EXAMPLE 2

$X \sim \mathrm{B}(150, 0.4)$. Use a normal approximation to find
(i) $\mathrm{P}(X \leqslant 55)$ (ii) $\mathrm{P}(X > 66)$ (iii) $\mathrm{P}(52 \leqslant X < 64)$ (iv) $\mathrm{P}(X = 60)$

Hint X takes whole numbers; Y is continuous. The whole number k is the interval from $k - 0.5$ to $k + 0.5$. Write the whole numbers you want, then include or leave out each end interval.

B2 $X \sim \mathrm{B}(200, 0.45)$. State a suitable normal approximation, then use it to find each probability.

(a) $\mathrm{P}(X < 85)$ [2 marks]

Answer .....

(b) $\mathrm{P}(X \geqslant 100)$ [2 marks]

Answer .....

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(c) $\mathrm{P}(80 < X \leqslant 95)$ [2 marks]

Answer .....

Total for B2: 6 marks

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EXAMPLE 3

48% of a type of seed grow into tall plants. 120 seeds are planted. X is the number that grow into tall plants.
(i) Find the exact value of $\mathrm{P}(X = 60)$.
(ii) Use a normal approximation to find $\mathrm{P}(X = 60)$.
(iii) Find the percentage error in the approximation.

Hint Percentage error $= \frac{|\text{approximate} - \text{exact}|}{\text{exact}} \times 100$. Keep more figures than you will give until the last line.

B3 In a town, 52% of voters support a plan. A random sample of 180 voters is taken. X is the number who support the plan.

(a) Find the exact value of $\mathrm{P}(X \leqslant 90)$. [2 marks]

Answer .....

(b) Use a normal approximation to find $\mathrm{P}(X \leqslant 90)$. [2 marks]

Answer .....

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(c) Find the percentage error in the approximation. Use at least 5 significant figures in your working. [2 marks]

EXAMPLE 4

$X \sim \mathrm{B}(240, 0.55)$. Use a normal approximation to find
(i) the largest value of k such that $\mathrm{P}(X < k) < 0.05$
(ii) the smallest value of m such that $\mathrm{P}(X > m) < 0.01$.

Hint Write the event with a continuity correction, use the inverse normal to find the boundary for Y, then step back to whole numbers. Check the values either side.

B4 $X \sim \mathrm{B}(400, 0.45)$. Use a normal approximation to find the smallest value of m such that $\mathrm{P}(X \geqslant m) < 0.05$. [5 marks]

Answer .....

Total for B3: 6 marks

Answer .....

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EXAMPLE 5

Over several years, 40% of the visitors to a library have borrowed a book. After a new layout, a random sample of 150 visitors contains 72 who borrow a book. Use a suitable approximation to test, at the 5% level of significance, whether the proportion who borrow a book has increased.

Hint Do the six steps as in the binomial tests. Only Step 4 changes: the binomial is replaced by a normal distribution, with a continuity correction.

B5 A shop says 55% of its customers choose the mint flavour. After a price change, a random sample of 220 customers contains 108 who choose mint. Use a normal approximation to test, at the 5% level, whether the proportion has decreased. Do the test one step at a time.

(a) Step 1 Model: define X, state its distribution if H_0 is true, and the normal approximation. [1 mark]

Answer .....

(b) Step 2 Hypotheses. [1 mark]

Answer .....

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(c) Step 3 Level and tails. [1 mark]

Answer .....

(d) Step 4 Calculate assuming H_0, with a continuity correction. [2 marks]

Answer .....

(e) Step 5 Compare. [1 mark]

Answer .....

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(f) Step 6 Conclude, in context. [1 mark]

C Practice

A-Level

C1 $X \sim \mathrm{B}(320, 0.45)$. Lily uses a normal approximation to find $\mathrm{P}(X > 135)$. She writes "$\mathrm{P}(Y > 134.5) = 0.8571$". Explain her mistake and find the correct approximation. [4 marks]

Total for B5: 7 marks

Answer .....

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C2 $X \sim \mathrm{B}(180, 0.5)$. Ravi writes "$X \approx Y \sim \mathrm{N}(90, 45)$, so $\mathrm{P}(X \leqslant 80) = \mathrm{P}\left(Z < \frac{80.5-90}{45}\right) = 0.4164$." State why the approximation is suitable, explain his error and find the correct value. [4 marks]

Answer .....

C3 0.8% of the tiles made by a machine are cracked. A box holds 600 tiles. X is the number of cracked tiles in the box. Ella uses a normal approximation to find the probability that no tile in the box is cracked.

(a) Find the exact probability that no tile is cracked. [2 marks]

Answer .....

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(b) Find Ella's approximation. [2 marks]

Answer .....

(c) Explain why her approximation is poor. [1 mark]

Total for C3: 5 marks

C4 In the past, 30% of the parts made by a factory needed reworking. A new process is used. In a random sample of 400 parts, 98 need reworking. Use a normal approximation to test, at the 1% level, whether the proportion has decreased. Do one step in each part.

(a) Step 1 Model and Step 2 Hypotheses. [2 marks]

Answer .....

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(b) Step 3 Level and tails, and Step 4 Calculate assuming H_0. [2 marks]

Answer .....

(c) Step 5 Compare and Step 6 Conclude. [2 marks]

Total for C4: 6 marks

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C5 A coin is biased so that $\mathrm{P}(\text{head}) = 0.47$. It is tossed 200 times. The question asks for a normal approximation to the probability that fewer than half of the tosses are heads. Tom works out $\mathrm{P}(X \leqslant 100)$ using the binomial distribution. Explain his two mistakes, and find the approximation. [4 marks]

D Exam-style questions

Edexcel exam style

D1 In a large town, 42% of the adults have a library card.

(a) A random sample of 30 adults is taken. Find the probability that at most 10 of them have a library card. [2 marks]

Answer .....

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(b) A second random sample of 300 adults is taken. Explain why a normal approximation can be used for the number in this sample with a library card. [2 marks]

(c) Use a normal approximation to find the probability that between 115 and 135 adults, inclusive, in the sample of 300 have a library card. [3 marks]

Answer .....

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(d) Five independent samples of 300 adults are taken. Find the probability that at least 4 of them have between 115 and 135 adults, inclusive, with a library card. [3 marks]

Answer .....

Total for D1: 10 marks

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D2

Of the people who buy from a website, 60% buy again within a year. After a redesign, the owner thinks this proportion has fallen. In a random sample of 250 buyers, 138 buy again. Sam uses a normal approximation and writes:
"$\mathrm{H}_0 : X = 150$, $\mathrm{H}_1 : X < 150$. $Y \sim \mathrm{N}(150, 60)$. $\mathrm{P}(Y < 138) = \mathrm{P}\left(Z < \frac{138-150}{60}\right) = 0.4207$. $0.4207 > 0.05$, so accept H_0. The proportion is still 60%."

Sam's working has several errors.

(a) Describe four errors in Sam's working. [4 marks]

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(b) Carry out the test correctly, at the 5% level. [6 marks]

Answer .....

Total for D2: 10 marks

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D3 A survey showed that 35% of the workers at a large company cycle to work. After a new car park opens, a random sample of 180 workers is taken and 74 of them cycle to work. Using a suitable approximation, test at the 5% level of significance whether the proportion of workers who cycle to work has changed. [7 marks]

Answer .....

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D4 $X \sim \mathrm{B}(90, 0.5)$.

(a) Find the exact value of $\mathrm{P}(X = 50)$. [2 marks]

Answer .....

(b) Use a normal approximation to find $\mathrm{P}(X = 50)$. [3 marks]

Answer .....

(c) Find the percentage error in your approximation. Use at least 5 significant figures in your working. [2 marks]

Answer .....

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(d) Explain why the same method would give a much larger percentage error for $X \sim \mathrm{B}(90, 0.1)$. [2 marks]

Total for D4: 9 marks

D5 A bakery says that 45% of its customers buy bread. The manager thinks the proportion is higher on Saturdays. She will ask a random sample of 200 Saturday customers.

(a) Write suitable hypotheses for a test of the manager's belief. [2 marks]

Answer .....

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(b) Use a normal approximation to find the critical region for the test at the 5% level. [4 marks]

Answer .....

(c) Use the binomial distribution to find the actual significance level of this critical region. [2 marks]

Answer .....

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(d) 101 customers in the sample buy bread. Comment on this. [2 marks]

Total for D5: 10 marks

D6 A seed packet says that 60% of the seeds will germinate. A gardener sows 250 seeds, to test at the 1% level whether the proportion that germinate is different from 60%.

(a) Give two reasons why a normal approximation is suitable for the number of seeds that germinate. [2 marks]

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(b) Use a normal approximation to find the critical region for the test. [4 marks]

Answer .....

(c) 128 of the seeds germinate. State the conclusion of the test in context. [2 marks]

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $X \sim \mathrm{B}(n, 0.4)$. Using a normal approximation with a continuity correction, $\mathrm{P}(X \geqslant 48) = 0.0629$, to 3 significant figures. Find n. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the mean and the variance of the approximating normal distribution in terms of n.

Hint 2 $X \geqslant 48$ becomes $Y > 47.5$. Find the z value with 0.0629 above it, and set up $\frac{47.5-\mu}{\sigma} = z$.

Hint 3 Let $u = \sqrt{n}$. Then $\mu = 0.4u^2$ and $\sigma = \sqrt{0.24}\,u$, so you have a quadratic in u.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can say when a normal approximation to B(n, p) is suitable and write it as N(np, np(1 – p)).	○ ○ ○
I can apply the continuity correction in the right direction for <, ≤, >, ≥ and =.	○ ○ ○
I can compare the approximation with the exact binomial and explain when it is poor.	○ ○ ○
I can use the approximation to find a boundary, a critical region or the result of a hypothesis test.	○ ○ ○

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