Choosing and carrying out the right test

138 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-choosing-and-carrying-out-the-right-test/

Date

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that the examiner reports say costs marks.

A1 Last lesson [2 marks]

$X \sim \mathrm{N}(80, 5^2)$. A random sample of 25 observations is taken. State the distribution of $\bar{X}$ and find $\mathrm{P}(\bar{X} \geqslant 81.8)$.

Answer .....

A2 Needed today [3 marks]

Name the parameter that the hypotheses are written in for each test. (a) Whether the proportion of faulty phones has fallen. (b) Whether the mean mass of a bag of flour has changed. (c) Whether there is positive correlation between two variables.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

A3 Common slip [2 marks]

Ali tests whether the proportion p of left-handed people in a club has changed from 0.4, at the 5% level. In a sample of 30, there are 17 left-handed people. He finds $\mathrm{P}(X \geqslant 17) = 0.04811$ and writes "$0.04811 < 0.05$, so reject H_0." Correct his comparison and his decision.

Answer .....

A4 Needed today [2 marks]

For a sample of 14 pairs, $r = 0.481$. For $n = 14$, the critical value for 5% in one tail is 0.4575. Test for positive correlation at the 5% level: state the decision.

Answer .....

A5 Earlier lesson [2 marks]

$X \sim \mathrm{B}(120, 0.35)$. Use a normal approximation to find $\mathrm{P}(X \geqslant 50)$.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

B Example, then your turn

A-Level

Every test has the same six steps. Step 1 Model: say what is measured or counted, and its distribution if H_0 is true. Step 2 Hypotheses: H_0 and H_1 in the parameter p, μ or ρ, never in the sample value. Step 3 Level and tails: the significance level, and one tail or two. Step 4 Calculate assuming H_0: the probability of the observed value or a more extreme one, or the critical region, from the model you chose. Step 5 Compare: that probability with the level, or the observed value with the critical region. Step 6 Conclude: reject or do not reject H_0, then one sentence in context. First decide what is recorded: a count of successes (p), a mean of measurements (μ) or pairs of values (ρ). The worked answers label every step.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 1

For each investigation, choose the test. State the model, and the parameter the hypotheses are written in.

(i) A shop says 30% of its customers pay by phone. In a random sample of 40 customers, 18 pay by phone. Is the proportion higher?

(ii) Bread rolls from a bakery have masses, in grams, that are normally distributed with standard deviation 4. The mean mass used to be 65 g. The mean mass of a random sample of 20 rolls is found. Has the mean changed?

(iii) The heights and arm spans of a random sample of 12 adults are measured, and the PMCC is 0.64. Is there positive correlation?

Hint Ask what is recorded. A count of successes in a fixed number of trials: binomial, parameter p. A mean of measurements with known standard deviation: normal, parameter μ. Pairs of values: PMCC, parameter ρ.

B1 For each investigation, state the model, and the hypotheses in the right parameter.

(a) A polling company says 52% of voters support a plan. In a random sample of 500 voters, 243 support it. Has support fallen? [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) For a random sample of 25 days, the PMCC between the daily rainfall and the number of visitors to a park is found. Is there any correlation? [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

A gardener believes that tomato plants that get more sunlight grow taller. For a random sample of 14 plants, the PMCC between the hours of sunlight per day and the height of the plant is 0.583. For $n = 14$, the critical value for 1% in one tail is 0.6120. Test the gardener's belief at the 1% level.

Hint The data are pairs, so the test is on ρ. "Taller with more sunlight" is positive correlation: one tail, upper.

B2 A car magazine believes that cars with larger engines travel fewer miles per gallon. For a random sample of 18 cars, the PMCC between the engine size and the miles per gallon is -0.553. For $n = 18$, the critical value for 1% in one tail is 0.5425. Test the magazine's belief at the 1% level. Do the test one step at a time.

(a) Step 1 Model. [1 mark]

Answer .....

(b) Step 2 Hypotheses. [1 mark]

Answer .....

Total for B1: 6 marks

(c) Batteries last a time, in hours, that is normally distributed with standard deviation 6; the mean should be 90. The mean of a random sample of 30 batteries is found. Is the mean less than 90? [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Step 3 Level and tails. [1 mark]

Answer .....

(d) Step 4 Calculate assuming H_0: the critical region. [1 mark]

Answer .....

(e) Step 5 Compare. [1 mark]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 3

A supermarket says that 42% of its customers use the self-checkout. After the tills are changed, 141 of a random sample of 300 customers use the self-checkout. Test, at the 5% level, whether the proportion has changed.

Hint A count of successes: binomial, parameter p. With $n = 300$ and p near 0.5, use a normal approximation with a continuity correction. "Changed": two tails.

B3 A charity says 55% of people asked in the street give money. A new volunteer asks a random sample of 160 people, and 76 give money. Test, at the 5% level, whether the proportion who give to this volunteer is lower than 55%.

(a) Steps 1 to 3: the model (with the approximation), the hypotheses, and the level and tail. [3 marks]

(f) Step 6 Conclude, in context. [2 marks]

Total for B2: 7 marks

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Steps 4 to 6. [3 marks]

Total for B3: 6 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

Bricks from a factory have masses, in kg, that are normally distributed with mean 3.00 and standard deviation 0.15. The factory also says 8% of its bricks are cracked. After a new kiln is fitted:
(i) a random sample of 25 bricks has mean mass 2.94 kg; test at the 5% level whether the mean mass has decreased;
(ii) a random sample of 40 bricks has no cracked bricks; test at the 5% level whether the proportion of cracked bricks has decreased.

Hint Two different quantities, so two different tests. Masses are measurements with a known standard deviation: test μ. Cracked bricks are counted: test p with an exact binomial.

B4 Fish in a lake have lengths, in cm, that are normally distributed with mean 38 and standard deviation 4.2. Of the fish, 30% are female. After a dry year a biologist takes random samples.

(a) A sample of 20 fish has mean length 39.9 cm. Assuming the standard deviation is unchanged, test at the 5% level whether the mean length has changed. [4 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) A sample of 25 fish has 12 females. Test at the 5% level whether the proportion of females has increased. [4 marks]

Total for B4: 8 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 5

A teacher records the hours of revision and the test mark for a random sample of 9 students. The PMCC is 0.71. She asks whether there is any correlation between revision and mark. For $n = 9$, the critical values for one tail are 0.5822 (5%) and 0.6664 (2.5%).

(i) Explain why a PMCC test is the right test, and not a binomial or normal-mean test.

(ii) Carry out the test at the 5% level.

Hint Each student gives a pair of values; nothing is counted out of a fixed number of trials, and no mean is compared with a known value. "Any correlation" is two-tailed: use the 2.5% column.

B5 Ria thinks a die is biased towards six. She throws it 30 times and gets 10 sixes. Choose a suitable test, and carry it out at the 5% level. [7 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 For a random sample of 14 towns, the PMCC between the height above sea level and the yearly rainfall is 0.521. For $n = 14$, the critical values for one tail are 0.4575 (5%) and 0.5324 (2.5%). Test, at the 5% level, whether there is any correlation between height and rainfall. Do the steps in the parts.

(a) Step 1 Model and Step 2 Hypotheses. [2 marks]

Answer .....

(b) Step 3 Level and tails, and Step 4 Calculate assuming H_0: the critical region. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Step 5 Compare and Step 6 Conclude. [2 marks]

Total for C1: 6 marks

C2 A factory says 10% of its plates have a fault. A manager thinks the proportion has increased. In a random sample of 50 plates, 9 have a fault. Lena uses $Y \sim \mathrm{N}(5, 4.5)$ and finds $\mathrm{P}(Y > 8.5) = 0.04948 < 0.05$, so she rejects H_0. Explain why the normal approximation is not suitable here, and carry out the test at the 5% level with a suitable distribution. [5 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C3 A random sample of 10 pairs of values has a PMCC of 0.52. Omar tests for positive correlation at the 5% level. He uses the table row for $n = 20$, where the critical value is 0.3783, and rejects H_0. For $n = 10$, the critical value for 5% in one tail is 0.5494.

(a) Explain Omar's error. [2 marks]

(b) Carry out the test correctly. [4 marks]

Total for C3: 6 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 A type of light bulb lasts a time, in hours, that is normally distributed with mean 1200 and standard deviation 50. A supplier changes the filament. A random sample of 40 new bulbs has mean lifetime 1183 hours. Choose a suitable test and carry it out at the 5% level to decide whether the mean lifetime has changed. Assume the standard deviation is unchanged. [6 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C5 For each investigation, say which test should be used: an exact binomial test, a binomial test with a normal approximation, a test of a normal mean, or a PMCC test. Give a reason.

(a) A coin is tossed 400 times to see whether it is biased. A random sample of 12 students give their height and shoe size, to test for positive correlation. [2 marks]

(b) In a sample of 500 parts, a factory counts how many are faulty; the claimed rate is 1%. A baker weighs a random sample of 30 cakes to test whether the mean mass has fallen; the masses are known to be normally distributed with standard deviation 8 g. [2 marks]

Total for C5: 4 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D1 A running club records, for a random sample of 12 members, the hours they train each week and their time for a 5 km run. The PMCC is -0.632. For $n = 12$, the critical value for 5% in one tail is 0.4973.

(a) Test, at the 5% level, whether members who train for longer have shorter run times. [4 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) The club's run times, in minutes, are normally distributed with mean 24 and standard deviation 1.8. After a new coach starts, a random sample of 15 members has mean time 23.1 minutes. Assuming the standard deviation is unchanged, test at the 1% level whether the mean time has decreased. [5 marks]

(c) A member says, "Part (a) shows that training more makes you faster." Comment on this. [2 marks]

Total for D1: 11 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D2

> A cinema says that 56% of its tickets are bought online. After a new app is launched, the manager thinks the proportion has increased. In a random sample of 250 tickets, 154 were bought online. Jake writes: "$\mathrm{H}_0 : p = 0.56$, $\mathrm{H}_1 : p \neq 0.56$. $Y \sim \mathrm{N}(140, 61.6)$. $\mathrm{P}(Y \geqslant 154) = 0.03723$. $0.03723 > 0.025$, so do not reject H_0. This proves the app has made no difference."

Jake's working has errors.

(a) Describe three errors in Jake's working. [3 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Carry out the test correctly at the 5% level. [7 marks]

Answer .....

Total for D2: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D3 A bank says that 15% of its customers pay with cash. A new branch opens. In a random sample of 40 customers at the new branch, 2 pay with cash. Test, at the 10% level of significance, whether the proportion of customers who pay with cash at the new branch is different from 15%. [7 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D4 Bags of compost should hold a mean of 100 litres. The volumes are normally distributed with standard deviation 3 litres. A random sample of 20 bags has mean volume 98.8 litres.

(a) Find the critical region for $\bar{X}$ for a test of whether the mean volume is different from 100 litres at the 5% level, and state the conclusion. [4 marks]

(b) A customer group suspects the bags are under-filled. Find the critical region for a test of whether the mean volume is less than 100 litres at the 5% level, and state the conclusion. [3 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Explain why the hypotheses must be chosen before the sample is taken. [2 marks]

Total for D4: 9 marks

D5 The proportion of a town's adults who own a bicycle is thought to be 0.45. A council suspects the proportion is lower.

(a) A random sample of 20 adults is taken. Find the critical region for a test at the 5% level, and state the actual significance level. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) In a larger random sample of 400 adults, 162 own a bicycle. Test the council's suspicion at the 5% level. [5 marks]

(c) Explain why a normal approximation is suitable in (b) but was not used in (a). [2 marks]

Total for D5: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D6 A machine cuts rods. Their lengths, in cm, are normally distributed with mean 20 and standard deviation 0.3. A rod is too long if its length is over 20.5 cm.

(a) After a repair, a random sample of 25 rods has mean length 20.12 cm. Assuming the standard deviation is unchanged, test at the 5% level whether the mean length has changed. [4 marks]

(b) Find the probability that one rod is too long, when the mean is 20. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) A second random sample of 50 rods has 6 that are too long. Using your answer to (b), test at the 5% level whether the proportion of rods that are too long has increased. [3 marks]

Total for D6: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

No route is given. The hints are at the end of the sheet. Use one at a time.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E1 A machine fills tins whose masses, in grams, are normally distributed with standard deviation 4. Each day the mean mass $\bar{x}$ of a random sample of 9 tins is used to test $\mathrm{H}_0 : \mu = 500$ against $\mathrm{H}_1 : \mu < 500$ at the 5% level. On each of 40 days the machine is set correctly, so $\mu = 500$, but H_0 is rejected on 6 of the days. A manager thinks the daily test rejects H_0 too often. Test the manager's belief at the 5% level. Then find the value of μ for which the daily test rejects H_0 with probability 0.15. [10 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 First find the value of $\bar{x}$ below which the daily test rejects H_0.

Hint 2 When $\mu = 500$ each day has probability 0.05 of a rejection, so the number of rejections in 40 days is binomial.

Hint 3 For the last part the critical value stays the same; only μ changes. Use $\mathrm{P}(Z < z) = 0.15$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can choose the model and the parameter from the context: p for a proportion, μ for a mean, ρ for pairs of values.	○ ○ ○
I can decide whether a binomial test needs the exact distribution or a normal approximation.	○ ○ ○
I can carry out a binomial, normal-mean or PMCC test with the six steps, one-tailed or two-tailed.	○ ○ ○
I can write a conclusion in context and explain why a test or an approximation is suitable.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.