Conditional probability and Venn diagrams

100 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-conditional-probability-and-venn-diagrams/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks in the exam.

A1 Needed today [3 marks]

A club has 80 members. 35 are adults. 12 of the adults and 20 of the children swim. Find the probability that a member chosen at random swims, and the probability that a member chosen at random is an adult who swims.

Answer .....

A2 Needed today [2 marks]

$\mathrm{P}(A) = 0.45$, $\mathrm{P}(B) = 0.3$ and $\mathrm{P}(A \cap B) = 0.1$. Find $\mathrm{P}(A \cap B')$ and $\mathrm{P}(A' \cap B')$.

Answer .....

A3 Needed today [2 marks]

Write $\dfrac{0.18}{0.45}$ and $\dfrac{0.12}{0.32}$ as fractions in their lowest terms.

Answer .....

A4 Common slip [2 marks]

Two fair six-sided dice are rolled. Find the probability that the total is 8, and the probability that at least one die shows a 2.

Answer .....

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PART 1 Conditional probability from a table or sample space

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EXAMPLE 1 · I DO

The table shows 160 visitors to a museum. A visitor is chosen at random. (a) Find the probability that the visitor bought a guide, given that the visitor is an adult. (b) Find the probability that the visitor is a child, given that the visitor bought a guide. (c) Find $\mathrm{P}(\text{bought a guide} \mid \text{child})$. (d) Determine whether being an adult and buying a guide are independent.

	Bought a guide	No guide
Adult	42	58
Child	24	36

Hint "Given that" tells you which group you are in. That group's total is the denominator; the numerator is the people in the group who also have the other property.

YOUR TURN · WE DO

B1 The table shows the 240 workers at a company, by where they work and whether they travel to work by train. A worker is chosen at random.

	Train	Not train
Office	36	54
Site	18	72
Home	6	54

(a) Find $\mathrm{P}(\text{train} \mid \text{office})$. [1 mark]

Answer .....

(b) Find the probability that the worker works on site, given that the worker travels by train. [1 mark]

Answer .....

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(c) Find the probability that the worker does not work in the office, given that the worker does not travel by train. [2 marks]

Answer .....

(d) Determine whether "works from home" and "travels by train" are independent. [2 marks]

Answer .....

Total for B1: 6 marks

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EXAMPLE 2 · I DO

A fair spinner numbered 1 to 4 and a fair spinner numbered 1 to 3 are spun. (a) Draw a sample-space diagram of the product of the two scores. (b) Given that the product is even, find the probability that the total of the two scores is 5 or more. (c) Given that the total is 5 or more, find the probability that the product is even.

Hint Mark the outcomes in the "given" event first. They are the new sample space: count them, then count the ones that also have the other property.

YOUR TURN · WE DO

B2 A fair six-sided dice and a fair spinner numbered 1 to 5 are rolled and spun, and the two scores are added.

(a) Find the number of outcomes in which at least one of the scores is 5. [2 marks]

Answer .....

(b) Given that at least one of the scores is 5, find the probability that the total is 9 or more. [2 marks]

Answer .....

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(c) Given that the total is 9 or more, find the probability that at least one of the scores is 5. [2 marks]

Answer .....

Total for B2: 6 marks

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C1 A company uses two couriers, X and Y. The table shows 120 parcels. Ravi says: "The probability that a parcel is late, given that it went with courier X, is $\frac{14}{30}$." Explain Ravi's mistake and find the correct probability. Which courier is more likely to deliver a parcel late? Give a reason. [5 marks]

	Late	On time
Courier X	14	56
Courier Y	16	34

Answer .....

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C2 Two fair spinners, each numbered 1 to 5, are spun. Dev says: "Given that at least one spinner shows a 3, the probability that both show a 3 is $\frac{1}{5}$, because the other spinner shows a 3 one time in five." Use a sample space to show that Dev is wrong. Find the correct probability, and the probability that the total is 7 or more, given that at least one spinner shows a 3. [6 marks]

Answer .....

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PART 2 Conditional probability from a Venn diagram

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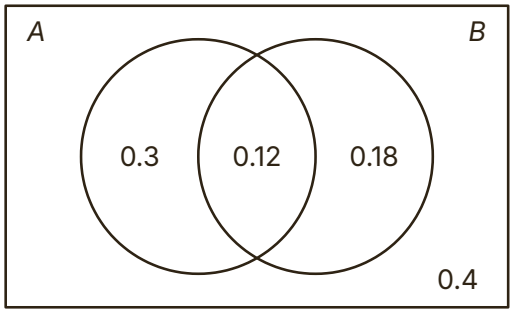
EXAMPLE 3 · I DO

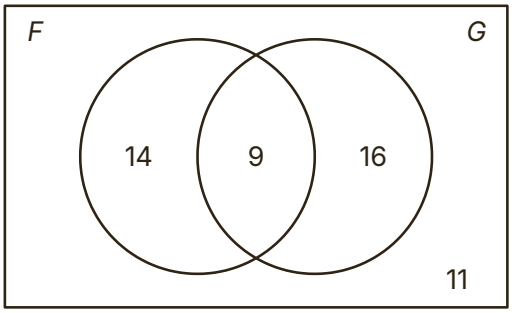
The Venn diagram shows the probabilities for two events A and B. Find (a) $\mathrm{P}(A \mid B)$, (b) $\mathrm{P}(B \mid A)$, (c) $\mathrm{P}(B \mid A \cup B)$, (d) $\mathrm{P}(A' \mid B')$.



Hint $\mathrm{P}(X \mid Y) = \dfrac{\mathrm{P}(X \cap Y)}{\mathrm{P}(Y)}$. Shade Y first; the numerator is the part of Y that is also in X.

YOUR TURN · WE DO

B3 50 people were asked whether they have been to France (F) and whether they have been to Germany (G). The Venn diagram shows the numbers. A person is chosen at random. Find



(a) $\mathrm{P}(F \mid G)$ [1 mark]

Answer .....

(b) $\mathrm{P}(G' \mid F)$ [1 mark]

Answer .....

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(c) $\mathrm{P}(F \mid F \cup G)$ [2 marks]

Answer .....

(d) $\mathrm{P}(G \mid F')$ [2 marks]

Answer .....

Total for B3: 6 marks

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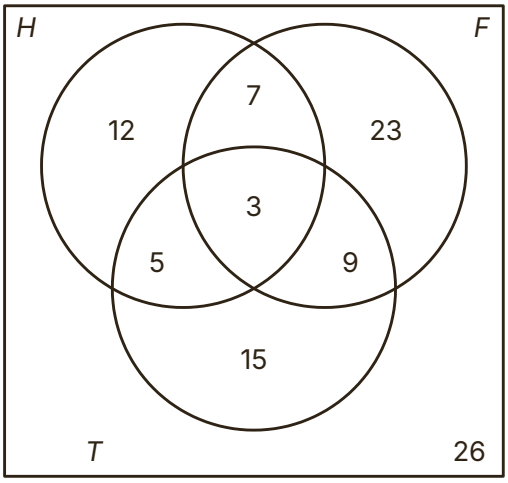
EXAMPLE 4 · I DO

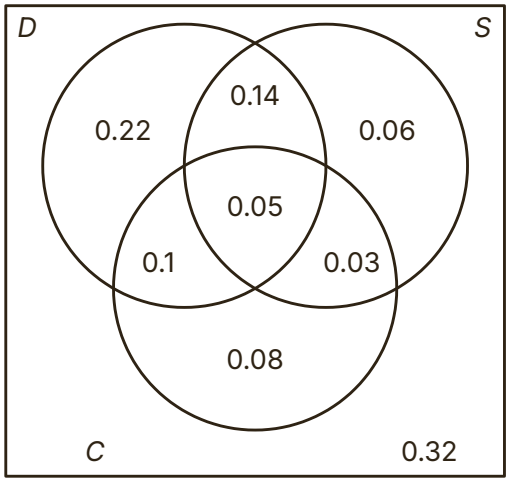
100 students were asked whether they play hockey (H), football (F) or tennis (T). The Venn diagram shows the numbers. A student is chosen at random. Find (a) $\mathrm{P}(H \mid F)$, (b) $\mathrm{P}(F \cap T \mid H)$, (c) the probability that the student plays exactly one sport, given that the student plays at least one.



Hint For "given at least one", the people outside every circle are not in the group, so they leave the denominator.

YOUR TURN · WE DO

B4 The Venn diagram shows the probabilities that a customer at a café buys a drink (D), a sandwich (S) or a cake (C). A customer is chosen at random. Find



(a) $\mathrm{P}(C \mid S)$ [2 marks]

Answer .....

(b) $\mathrm{P}(D \mid S \cup C)$ [2 marks]

Answer .....

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(c) the probability that the customer buys exactly two of the items, given that the customer buys a drink [2 marks]

Answer .....

Total for B4: 6 marks

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C3 Of 75 teenagers, 34 own a bike, 21 own a scooter and 9 own both. One of them is chosen at random. Leah writes: "Given that the teenager owns at least one of the two, the probability that they own both is $\frac{9}{75}$." Explain Leah's mistake. Find the correct probability, and the probability that the teenager owns a scooter but not a bike, given that they own at least one of the two. [5 marks]

Answer .....

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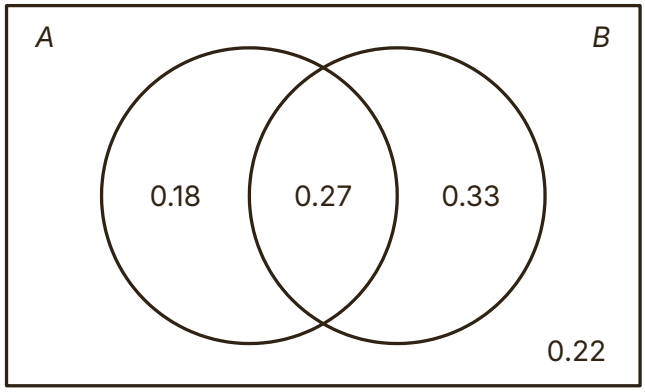
C4 The Venn diagram shows the probabilities for two events A and B. [5 marks]

Nia writes: "$\mathrm{P}(B \mid A) = \dfrac{0.27}{0.6} = 0.45$." Explain Nia's mistake. Find $\mathrm{P}(B \mid A)$, and use conditional probability to determine whether A and B are independent.



Answer .....

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PART 3 Independent events

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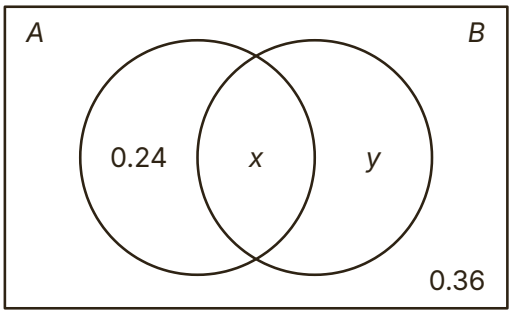
EXAMPLE 5 · I DO

$\mathrm{P}(A) = 0.4$, $\mathrm{P}(B \mid A) = 0.35$ and $\mathrm{P}(A \cup B) = 0.6$. (a) Find $\mathrm{P}(A \cap B)$ and $\mathrm{P}(B)$. (b) Find $\mathrm{P}(A \mid B)$. (c) Determine whether A and B are independent.

Hint $\mathrm{P}(B \mid A)$ is the fraction of A that is also in B, so $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B \mid A)$.

YOUR TURN · WE DO

B5 The Venn diagram shows the probabilities for two events A and B, where x and y are probabilities. $\mathrm{P}(B \mid A) = 0.4$.



(a) Find x and y. [3 marks]

Answer .....

(b) Show that A and B are independent. [1 mark]

Answer .....

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(c) Find $\mathrm{P}(A \mid B')$. [2 marks]

Answer .....

Total for B5: 6 marks

PRACTICE · YOU DO

C5 For two events, $\mathrm{P}(A) = 0.5$ and $\mathrm{P}(B \mid A) = 0.3$. Tom writes: "$\mathrm{P}(A \cap B) = 0.3$." Explain Tom's mistake and find $\mathrm{P}(A \cap B)$. Given also that $\mathrm{P}(B) = 0.25$, find $\mathrm{P}(A \mid B)$ and $\mathrm{P}(A \cup B)$. [5 marks]

Answer .....

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C6 For two events A and B, $\mathrm{P}(A) = 0.6$, $\mathrm{P}(B \mid A) = 0.45$ and $\mathrm{P}(A' \mid B') = 0.5$. Find $\mathrm{P}(B)$ and $\mathrm{P}(A \mid B)$, and state, with a reason, whether A and B are independent. [8 marks]

Answer .....

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D1 A test for a medical condition was given to 250 people. The table shows the results.

	Positive	Negative
Has the condition	46	4
Does not have it	18	182

(a) Find the probability that the test is positive, given that the person has the condition, and the probability that the test is positive, given that the person does not have the condition. [3 marks]

Answer .....

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(b) Find the probability that a person has the condition, given that the test is positive. [3 marks]

Answer .....

(c) Find the probability that a person does not have the condition, given that the test is negative. [2 marks]

Answer .....

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(d) A doctor says: "This test is positive 92% of the time for people with the condition, so a person with a positive result has a 92% chance of having it." Comment on the doctor's statement. [2 marks]

Total for D1: 10 marks

D2

> $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.5$ and $\mathrm{P}(A \cup B) = 0.8$. Zoe writes: "$A$ and B have nothing to do with each other, so they are independent and $\mathrm{P}(A \mid B) = \mathrm{P}(A) = 0.3$."

Zoe is wrong.

(a) Show that A and B are mutually exclusive. [2 marks]

Answer .....

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(b) Find $\mathrm{P}(A \mid B)$, and explain why A and B are not independent. [3 marks]

(c) A third event C has $\mathrm{P}(C) = 0.25$, $\mathrm{P}(A \mid C) = 0.6$ and $\mathrm{P}(B \mid C) = 0.4$. Find $\mathrm{P}(C \mid A)$ and $\mathrm{P}(C \mid A \cup B)$. [4 marks]

Answer .....

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Two events A and B have $\mathrm{P}(A \mid B) = 2\,\mathrm{P}(B \mid A)$, $\mathrm{P}(A' \mid B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.65$.

(a) Show that $\mathrm{P}(A) = 2\,\mathrm{P}(B)$. [2 marks]

Answer .....

(b) Find $\mathrm{P}(A)$, $\mathrm{P}(B)$ and $\mathrm{P}(A \cap B)$. [4 marks]

Answer .....

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(c) Find $\mathrm{P}(B \mid A')$. [2 marks]

Answer .....

Total for E1: 8 marks

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $\mathrm{P}(A) = a$, $\mathrm{P}(B) = b$ and $\mathrm{P}(A \cap B) = p$. Each conditional probability is p over something.

Hint 2 $\frac{p}{b} = \frac{2p}{a}$ gives a link between a and b. $\mathrm{P}(A' \mid B) = 0.6$ means $\mathrm{P}(A \mid B) = 0.4$, so $p = 0.4b$.

Hint 3 Put everything into $a + b - p = 0.65$ to get one equation in b.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find a conditional probability from a two-way table or a sample space. ○ ○ ○

I can find conditional probabilities from two-set and three-set Venn diagrams. ○ ○ ○

I can tell P(A | B) from P(B | A) and use the right denominator. ○ ○ ○

I can test independence with P(A | B) = P(A) and find unknowns from a conditional probability. ○ ○ ○

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