# Hypothesis testing for zero correlation

104 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-correlation-hypothesis-test/

Date



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## A Do Now

Quick questions to start: the six steps, the hypotheses in $\rho$, and reading the table.

**A1** Needed today [2 marks]

The six steps of a hypothesis test are listed in the wrong order: Compare; Hypotheses; Conclude; Model; Calculate assuming $\mathrm{H}_0$; Level and tails. Write them in the right order.

**A2** Common slip [2 marks]

Ola writes $\mathrm{H}_0 : r = 0$, $\mathrm{H}_1 : r > 0$ for a test of positive correlation. Explain what is wrong, and correct the hypotheses.

**A3** Needed today [2 marks]

Use the table of critical values of the PMCC. For a sample of size $12$, find the critical value for (a) a one-tailed test at the $5\%$ level, (b) a two-tailed test at the $5\%$ level.

| $n$ | $0.10$ | $0.05$ | $0.025$ | $0.01$ |
|---|---|---|---|---|
| $11$ | $0.4187$ | $0.5214$ | $0.6021$ | $0.6851$ |
| $12$ | $0.3981$ | $0.4973$ | $0.5760$ | $0.6581$ |
| $13$ | $0.3802$ | $0.4762$ | $0.5529$ | $0.6339$ |

Answer .....

**A4** Needed today [3 marks]

Write $\mathrm{H}_1$ for each question. (a) "Is there positive correlation between $x$ and $y$?" (b) "Is there any correlation between $x$ and $y$?" (c) "Do larger values of $x$ go with smaller values of $y$?"

Answer .....


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**A5** Common slip [2 marks]

A question asks, "Test whether there is correlation between $x$ and $y$." The sample PMCC is $-0.48$. Sam uses $\mathrm{H}_1 : \rho < 0$ because $r$ is negative. Explain why Sam is wrong.


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## PART 1 One-tailed tests


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**EXAMPLE 1 · I DO**

A café owner believes that on hotter days more cold drinks are sold. For a random sample of $14$ days, the PMCC between the daily maximum temperature and the number of cold drinks sold is $0.52$. Test the owner's belief at the $5\%$ level of significance.

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $13$ | $0.4762$ | $0.5529$ | $0.6339$ | $0.6835$ |
| $14$ | $0.4575$ | $0.5324$ | $0.6120$ | $0.6614$ |
| $15$ | $0.4409$ | $0.5140$ | $0.5923$ | $0.6411$ |

**Hint** Write each step on its own line. The hypotheses are about $\rho$, the PMCC of the population.

**YOUR TURN · WE DO**

**B1** A shop manager believes that weeks with more money spent on adverts have higher sales. For a random sample of $12$ weeks, the PMCC between the money spent on adverts and the weekly sales is $0.47$. Test her belief at the $5\%$ level of significance, in the steps below.

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $11$ | $0.5214$ | $0.6021$ | $0.6851$ | $0.7348$ |
| $12$ | $0.4973$ | $0.5760$ | $0.6581$ | $0.7079$ |
| $13$ | $0.4762$ | $0.5529$ | $0.6339$ | $0.6835$ |

**(a)** Step 1 Model. [1 mark]

Answer .....

**(b)** Step 2 Hypotheses. [2 marks]

Answer .....


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**(c)** Step 3 Level and tails. [1 mark]

Answer .....

**(d)** Step 4 Calculate assuming $H_0$: the critical value and the critical region. [2 marks]

Answer .....

**(e)** Step 5 Compare. [1 mark]

Answer .....


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**EXAMPLE 2 · I DO**

A geography student believes that higher places are colder. For a random sample of $20$ weather stations, the PMCC between the height above sea level and the mean temperature is $-0.41$. Test, at the $5\%$ level, whether there is negative correlation.

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $19$ | $0.3887$ | $0.4555$ | $0.5285$ | $0.5751$ |
| $20$ | $0.3783$ | $0.4438$ | $0.5155$ | $0.5614$ |
| $21$ | $0.3687$ | $0.4329$ | $0.5034$ | $0.5487$ |

**Hint** For $\mathrm{H}_1 : \rho < 0$ the critical region is $r < -(\text{table value})$.

**YOUR TURN · WE DO**

**B2** For a random sample of $16$ students, the PMCC between daily screen time and hours of sleep is $-0.39$. Test, at the $2.5\%$ level of significance, whether there is negative correlation between screen time and sleep.

**Total for B1: 8 marks**

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $15$ | $0.4409$ | $0.5140$ | $0.5923$ | $0.6411$ |
| $16$ | $0.4259$ | $0.4973$ | $0.5742$ | $0.6226$ |
| $17$ | $0.4124$ | $0.4821$ | $0.5577$ | $0.6055$ |

**(a)** Steps 1 and 2: Model and Hypotheses. [3 marks]

Answer .....

**(f)** Step 6 Conclude, in context. [1 mark]


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**(b)** Steps 3 and 4: Level and tails, and the critical region. [2 marks]

Answer .....

**(c)** Steps 5 and 6: Compare and Conclude. [2 marks]

**PRACTICE · YOU DO**

**C1** After a correlation test at the $5\%$ level with $\mathrm{H}_1 : \rho > 0$, four students write their results. Explain what is wrong with each. (1) "Reject $\mathrm{H}_0$. There is no evidence of correlation." (2) "Reject $\mathrm{H}_0$. This proves that there is positive correlation." (3) "$\mathrm{H}_0$ means the sample has no correlation." (4) "$r = 0.41$ is bigger than $0.05$, so reject $\mathrm{H}_0$." [4 marks]

**Total for B2: 7 marks**


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**C2** A random sample of $18$ shops is taken. The PMCC between the floor area of a shop and its weekly takings is $0.45$. Test, at the $2.5\%$ level of significance, whether there is positive correlation, in the steps below.

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $17$ | $0.4124$ | $0.4821$ | $0.5577$ | $0.6055$ |
| $18$ | $0.4000$ | $0.4683$ | $0.5425$ | $0.5897$ |
| $19$ | $0.3887$ | $0.4555$ | $0.5285$ | $0.5751$ |

**(a)** Steps 1 and 2: Model and Hypotheses. [2 marks]

Answer .....

**(b)** Steps 3 and 4: Level and tails, and the critical region. [2 marks]

Answer .....


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**(c)** Steps 5 and 6: Compare and Conclude. [3 marks]

**Total for C2: 7 marks**


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## PART 2 Two-tailed tests


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**EXAMPLE 3 · I DO**

A random sample of $30$ towns is taken. The PMCC between the distance from the coast and the mean house price is $-0.38$. Test, at the $5\%$ level, whether there is correlation between the distance from the coast and the mean house price.

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $29$ | $0.3115$ | $0.3673$ | $0.4297$ | $0.4705$ |
| $30$ | $0.3061$ | $0.3610$ | $0.4226$ | $0.4629$ |
| $31$ | $0.3009$ | $0.3550$ | $0.4158$ | $0.4556$ |

**Hint** Decide the tails from the WORDS of the question. Two tails at $5\%$ means $2.5\%$ in each tail.

**YOUR TURN · WE DO**

**B3** A random sample of $25$ apple trees is taken. The PMCC between the height of a tree and its yield is $0.43$. Test, at the $1\%$ level of significance, whether there is any correlation between height and yield.

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $24$ | $0.3438$ | $0.4044$ | $0.4716$ | $0.5151$ |
| $25$ | $0.3365$ | $0.3961$ | $0.4622$ | $0.5052$ |
| $26$ | $0.3297$ | $0.3882$ | $0.4534$ | $0.4958$ |

**(a)** Steps 1 to 3: Model, Hypotheses, and Level and tails. [3 marks]

Answer .....


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**(b)** Steps 4 to 6: the critical region, Compare and Conclude. [4 marks]

**Total for B3: 7 marks**


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**C3** A random sample of $26$ gardens is taken. The PMCC between the hours of sunshine a garden gets and the mass of tomatoes it grows is $-0.42$. Test, at the $5\%$ level of significance, whether there is correlation between the hours of sunshine and the mass of tomatoes. The table gives critical values of the PMCC for one tail. [6 marks]

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $24$ | $0.3438$ | $0.4044$ | $0.4716$ | $0.5151$ |
| $25$ | $0.3365$ | $0.3961$ | $0.4622$ | $0.5052$ |
| $26$ | $0.3297$ | $0.3882$ | $0.4534$ | $0.4958$ |


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**C4** For a random sample of $15$ days, the PMCC between the hours of sunshine and the number of umbrellas sold is $-0.62$. Leo is asked to test, at the $5\%$ level, whether there is correlation between them. He writes: "$\mathrm{H}_0 : \rho = -0.62$, $\mathrm{H}_1 : \rho < -0.62$. One-tailed. Critical value $0.4409$. $-0.62 < 0.4409$, so I do not reject $\mathrm{H}_0$." Identify Leo's mistakes, and carry out the test correctly. [7 marks]

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $14$ | $0.4575$ | $0.5324$ | $0.6120$ | $0.6614$ |
| $15$ | $0.4409$ | $0.5140$ | $0.5923$ | $0.6411$ |
| $16$ | $0.4259$ | $0.4973$ | $0.5742$ | $0.6226$ |


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## PART 3 Finding r and the sample size


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**EXAMPLE 4 · I DO**

A random sample of $8$ students record the hours of practice, $x$, and their score in a typing test, $y$. Test, at the $5\%$ level, whether there is positive correlation between the hours of practice and the typing score. For $n = 8$, the critical values of the PMCC for one tail are $0.5067$ ($10\%$), $0.6215$ ($5\%$), $0.7067$ ($2.5\%$), $0.7887$ ($1\%$) and $0.8343$ ($0.5\%$).

| $x$ | **3** | **5** | **6** | **8** | **9** | **11** | **12** | **14** |
|---|---|---|---|---|---|---|---|---|
| $y$ | 21 | 24 | 20 | 30 | 26 | 28 | 35 | 31 |

**Hint** Find $r$ from the data on your calculator first, to $4$ decimal places. Then the six steps.

**YOUR TURN · WE DO**

**B4** A random sample of $10$ phones gives the age, $x$ months, and the battery life, $y$ hours. For $n = 10$, the critical values of the PMCC for one tail are $0.4428$ ($10\%$), $0.5494$ ($5\%$), $0.6319$ ($2.5\%$), $0.7155$ ($1\%$) and $0.7646$ ($0.5\%$).

| $x$ | $y$ |
|---|---|
| 2 | 48 |
| 3 | 40 |
| 5 | 41 |
| 6 | 52 |
| 7 | 39 |
| 8 | 44 |
| 10 | 37 |
| 11 | 45 |
| 12 | 30 |
| 14 | 40 |

**(a)** Find the PMCC between $x$ and $y$. [2 marks]

Answer .....


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**(b)** Test, at the $2.5\%$ level of significance, whether there is negative correlation between the age of a phone and its battery life. [6 marks]

**Total for B4: 8 marks**


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**C5** The PMCC of a random sample is $0.45$. A one-tailed test for positive correlation is carried out at the $5\%$ level. Use the table to find the smallest sample size for which the test gives evidence of positive correlation. [4 marks]

| $n$ | 0.05 | 0.025 | 0.01 | 0.005 |
|---|---|---|---|---|
| 12 | 0.4973 | 0.5760 | 0.6581 | 0.7079 |
| 13 | 0.4762 | 0.5529 | 0.6339 | 0.6835 |
| 14 | 0.4575 | 0.5324 | 0.6120 | 0.6614 |
| 15 | 0.4409 | 0.5140 | 0.5923 | 0.6411 |
| 16 | 0.4259 | 0.4973 | 0.5742 | 0.6226 |

Answer .....


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**C6** For nine randomly chosen days, the daily mean temperature, $x$ °C, and the daily rainfall, $y$ mm, are recorded. Test, at the $5\%$ level of significance, whether there is correlation between temperature and rainfall. For $n = 9$, the critical values of the PMCC for one tail are $0.4716$ $(10\%)$, $0.5822$ $(5\%)$, $0.6664$ $(2.5\%)$, $0.7498$ $(1\%)$ and $0.7977$ $(0.5\%)$. [7 marks]

| $x$ | **12** | **15** | **18** | **20** | **23** | **25** | **28** | **31** | **34** |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 5.2 | 4.1 | 6 | 4.8 | 3.9 | 5.5 | 4.4 | 3.6 | 4.7 |


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**D1**

For a random sample of $24$ used cars of one model, the PMCC between the age and the price is $-0.43$. Kai tests, at the $1\%$ level, whether there is negative correlation. He writes: "Model: $X \sim \mathrm{B}(24, 0.43)$. $\mathrm{H}_0 : \rho = 0$, $\mathrm{H}_1 : \rho < 0$. $1\%$, one-tailed: critical value $0.4716$. $-0.43 < 0.4716$, so reject $\mathrm{H}_0$. There is no correlation between age and price."

Kai's test is wrong.

**(a)** Identify three errors in Kai's work. [4 marks]

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $23$ | $0.3515$ | $0.4132$ | $0.4815$ | $0.5256$ |
| $24$ | $0.3438$ | $0.4044$ | $0.4716$ | $0.5151$ |
| $25$ | $0.3365$ | $0.3961$ | $0.4622$ | $0.5052$ |


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**(b)** Carry out the test correctly. [6 marks]

**Total for D1: 10 marks**


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**D2** A teacher believes that students who use a vocabulary app for longer get higher scores in a vocabulary test. For a random sample of $12$ boys, the PMCC between the time spent on the app and the test score is $0.52$. For a random sample of $20$ girls, it is $0.35$. Test the teacher's belief at the $5\%$ level for the boys and for the girls separately, and comment on the results. [9 marks]

| $n$ | $0.05$ | $0.025$ | $0.01$ | $0.005$ |
|---|---|---|---|---|
| $12$ | $0.4973$ | $0.5760$ | $0.6581$ | $0.7079$ |
| $20$ | $0.3783$ | $0.4438$ | $0.5155$ | $0.5614$ |


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# E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** The table gives critical values of the PMCC for one tail at the $2.5\%$ and $0.5\%$ levels. A random sample of size $n$ has a PMCC of $-0.40$. A test for correlation at the $5\%$ level (two-tailed) gives evidence of correlation, but the same test at the $1\%$ level (two-tailed) does not. Find all the possible values of $n$. [9 marks]

| $n$ | 0.025 | 0.005 | $n$ | 0.025 | 0.005 |
|---|---|---|---|---|---|
| 22 | 0.4227 | 0.5368 | 34 | 0.3388 | 0.4357 |
| 23 | 0.4132 | 0.5256 | 35 | 0.3338 | 0.4296 |
| 24 | 0.4044 | 0.5151 | 36 | 0.3291 | 0.4238 |
| 25 | 0.3961 | 0.5052 | 37 | 0.3246 | 0.4182 |
| 26 | 0.3882 | 0.4958 | 38 | 0.3202 | 0.4128 |
| 27 | 0.3809 | 0.4869 | 39 | 0.3160 | 0.4076 |
| 28 | 0.3739 | 0.4785 | 40 | 0.3120 | 0.4026 |
| 29 | 0.3673 | 0.4705 | 41 | 0.3081 | 0.3978 |
| 30 | 0.3610 | 0.4629 | 42 | 0.3044 | 0.3932 |
| 31 | 0.3550 | 0.4556 | 43 | 0.3008 | 0.3887 |
| 32 | 0.3494 | 0.4487 | 44 | 0.2973 | 0.3843 |
| 33 | 0.3440 | 0.4421 | 45 | 0.2940 | 0.3801 |


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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** Two-tailed at $5\%$ uses the $0.025$ column; two-tailed at $1\%$ uses the $0.005$ column.

**Hint 2** At $5\%$: reject when $-0.40 < -(\text{critical value})$, so you need the critical value below $0.40$. At $1\%$: do NOT reject, so you need that critical value at or above $0.40$.

**Hint 3** The critical values get smaller as $n$ grows. Find where each column crosses $0.40$.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can write the hypotheses of a correlation test in terms of $\rho$, choosing one or two tails from the wording. | ○ ○ ○ |
| I can find the critical value from the table and write the critical region. | ○ ○ ○ |
| I can compare r with the critical region and write a conclusion in context. | ○ ○ ○ |
| I can find the sample sizes for which a value of r is significant. | ○ ○ ○ |


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