Exponential models and coded regression

95 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-exponential-models/

Date

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A Do Now

Quick questions to start: the laws of logarithms, undoing a log, and two slips from the examiner reports.

A1 Needed today [2 marks]

Given that $y = 5 \times 2^x$, show that $\log y = \log 5 + x \log 2$.

Answer .....

A2 Needed today [2 marks]

Data are coded with $Y = \log y$ and $X = \log x$, and the coded points lie close to a straight line. State whether the relationship between y and x is of the form $y = ax^n$ or $y = kb^x$. Give a reason.

A3 Needed today [2 marks]

Solve (a) $\log a = 0.35$ (b) $\log b = -0.12$. Give each answer to 4 significant figures.

Answer .....

A4 Common slip [3 marks]

Ben writes $10^{1.2+0.3x} = 10^{1.2} + 10^{0.3x}$. Correct his working, and write $10^{1.2+0.3x}$ in the form kb^x, giving k and b to 3 significant figures.

Answer .....

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A5 Common slip [2 marks]

Data are coded with $Y = \log y$ and $X = \log x$. The regression line is $Y = 0.6 + 2.4X$. Ali says that $y = ax^n$ with $n = 10^{2.4}$. Correct Ali, and find a and n.

Answer .....

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PART 1 Models of the form y = kb^x

EXAMPLE 1 · I DO

The mass, m grams, of a culture is recorded each day for $1 \leqslant t \leqslant 8$. The data are coded with $y = \log m$ and $x = t$. The regression line of y on x is $y = 0.85 + 0.12x$. Find a model of the form $m = kb^t$.

Hint Write $\log m = 0.85 + 0.12t$, then undo the log: $m = 10^{0.85} \times \left(10^{0.12}\right)^t$.

YOUR TURN · WE DO

B1 The value, £V, of a machine is recorded each year for $1 \leqslant t \leqslant 10$. The data are coded with $y = \log V$ and $x = t$. The regression line of y on x is $y = 3.2 - 0.05x$. Find a model of the form $V = kb^t$, giving k and b to 3 significant figures. [3 marks]

Answer .....

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C1 Work backwards from each model to the coded straight line. (a) $y = 3.2 \times 1.15^x$: give the line of $\log y$ on x. (b) $y = 250x^{-0.6}$: give the line of $\log y$ on $\log x$. Give constants to 3 significant figures. [4 marks]

Answer .....

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C2 Data are coded with $y = \log P$ and $x = t$, and the regression line is $y = 0.8 + 0.15x$. Ana writes: "$P = 10^{0.8+0.15t} = 10^{0.8} + 10^{0.15t}$, so $P = 6.31 + 1.41t$, a straight line with $b = 0.15$." Identify the two mistakes in Ana's work, and find the correct model of the form $P = kb^t$. [5 marks]

Answer .....

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PART 2 Interpreting an exponential model

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EXAMPLE 2 · I DO

The number of bacteria, N, in a dish is counted every hour for $1 \leqslant t \leqslant 12$, where t is the time in hours. The data are coded with $y = \log N$ and $x = t$. The regression line of y on x is $y = 1.84 + 0.0414x$. (a) Find a model of the form $N = kb^t$. (b) Interpret k and b in context. (c) Estimate the time at which the model gives 200 bacteria, and comment on its reliability.

Hint $N = 10^{1.84+0.0414t} = 10^{1.84} \times 10^{0.0414t}$. Powers that are added become a product, not a sum.

YOUR TURN · WE DO

B2 A cup of coffee cools. Its temperature is θ degrees above room temperature after t minutes, for $0 \leqslant t \leqslant 40$. The data are coded with $y = \log \theta$ and $x = t$. The regression line of y on x is $y = 1.85 - 0.0177x$.

(a) Find a model of the form $\theta = kb^t$, giving k and b to 3 significant figures. [3 marks]

Answer .....

(b) Interpret b in context. [2 marks]

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(c) Find the time at which the model gives a temperature 20 degrees above room temperature. [2 marks]

Answer .....

Total for B2: 7 marks

PRACTICE · YOU DO

C3 The growth rate, g (per hour), of a culture is measured at temperatures T °C, for $5 \leqslant T \leqslant 30$. The data are coded with $y = \log g$ and $x = T$. The regression line is $y = -0.35 + 0.068x$. Sam says, "The intercept is negative, so the model says the culture shrinks at 0 °C." Explain why Sam is wrong, and find a model of the form $g = kb^T$. [5 marks]

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C4 A video is posted online. The number of views, V, is recorded on day d for $1 \leqslant d \leqslant 14$. The data are coded with $y = \log V$ and $x = d$, and the regression line is $y = 2.71 + 0.155x$. Find a model of the form $V = kb^d$, interpret b in context, and find the day on which the model first gives more than $50\,000$ views. Give one reason why the model may not be suitable for predicting the number of views after 60 days. [8 marks]

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PART 3 Models of the form y = ax^n

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EXAMPLE 3 · I DO

The length, l cm, and the mass, m grams, of 12 fish of one species are recorded, for $15 \leqslant l \leqslant 45$. The data are coded with $x = \log l$ and $y = \log m$. The regression line of y on x is $y = -1.92 + 2.95x$. (a) Show that $m = al^n$, and find a and n. (b) Estimate the mass of a fish of length 30 cm.

Hint Put $x = \log l$ and $y = \log m$ back into the line, then use the laws of logs to collect everything on one side: $\log m - 2.95 \log l = -1.92$.

YOUR TURN · WE DO

B3 The time, T seconds, for a ball to roll down a ramp of length L metres is recorded for $1 \leqslant L \leqslant 6$. The data are coded with $X = \log L$ and $Y = \log T$. The regression line of Y on X is $Y = -0.25 + 0.5X$.

(a) State whether the relationship between T and L is of the form $T = aL^n$ or $T = kb^L$. [1 mark]

Answer .....

(b) Find the values of a and n. [3 marks]

Answer .....

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(c) Estimate the time for a ramp of length 4 m. [2 marks]

Answer .....

Total for B3: 6 marks

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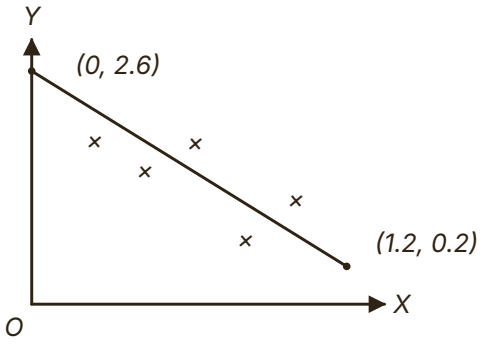
EXAMPLE 4 · I DO

The brightness, y units, of a lamp is measured at a distance of x metres, for $1 \leqslant x \leqslant 12$. The scatter diagram shows the coded data $X = \log x$ and $Y = \log y$, with the regression line of Y on X. The line passes through $(0, 2.6)$ and $(1.2, 0.2)$. Find a model of the form $y = ax^n$, and estimate the brightness at 5 m.



Hint The gradient of the coded line is n and the intercept on the Y-axis is $\log a$.

YOUR TURN · WE DO

B4 For $2 \leqslant x \leqslant 10$, the data are coded with $Y = \log y$ and $X = x$. The regression line of Y on X passes through the points $(2, 1.45)$ and $(10, 2.17)$. Find a model of the form $y = kb^x$, giving k and b to 3 significant figures, and use it to estimate y when $x = 6$. [5 marks]

Answer .....

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C5 Data are recorded for $5 \leqslant x \leqslant 50$ and coded with $X = \log x$ and $Y = \log y$. The regression line of Y on X is $Y = 1.3 + 0.75X$. Find a model of the form $y = ax^n$, and estimate y when $x = 20$. [5 marks]

Answer .....

C6 For trees of one kind with trunk diameter d cm, $10 \leqslant d \leqslant 80$, the height is h metres. The data are coded with $x = \log d$ and $y = \log h$, and the regression line is $y = 0.35 + 0.58x$.

(a) Find a model of the form $h = ad^n$. [2 marks]

Answer .....

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(b) Estimate the diameter of a tree of height 20 m. [2 marks]

Answer .....

(c) Ali says, "If the diameter doubles, the height doubles." Use the model to comment. [2 marks]

Total for C6: 6 marks

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D1 A patient is given a dose of a drug. The concentration, C mg per litre, of the drug in the blood is measured t hours after the dose. The data are coded with $y = \log C$ and $x = t$.

t (hours)	1	2	3	4	6	8
C (mg/l)	9.7	7.6	6.2	4.9	3.1	2.0

(a) Find the values of y, to 3 decimal places. [2 marks]

Answer .....

(b) Find the equation of the regression line of y on x. [2 marks]

Answer .....

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(c) Find a model of the form $C = kb^t$, giving k and b to 3 significant figures. [3 marks]

Answer .....

(d) Interpret k and b in context. [2 marks]

(e) A second dose is given when the concentration falls to 1 mg/l. Use the model to estimate when this happens, and comment on the reliability of your estimate. [2 marks]

Total for D1: 11 marks

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D2

The time, y hours, that a battery lasts when it powers a lamp drawing a current of x amps is recorded for $0.5 \leqslant x \leqslant 3$. The data are coded with $X = \log x$ and $Y = \log y$, and the regression line is $Y = 0.94 - 1.35X$. Jo writes: "$a = \mathrm{e}^{0.94} = 2.56$ and $n = 10^{-1.35} = 0.0447$, so $y = 2.56x^{0.0447}$."

Jo's model is wrong.

(a) Identify the two errors in Jo's work. [2 marks]

(b) Find the correct model. [3 marks]

Answer .....

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(c) Estimate how long the battery lasts with a current of 2 amps. [2 marks]

Answer .....

(d) Find the current for which the model gives 12 hours. [3 marks]

Answer .....

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 An app is launched. The number of users, N, is recorded at the end of week t for $2 \leqslant t \leqslant 12$. The data are coded with $y = \log N$ and $x = t$, and the regression line of y on x is $y = 2.16 + 0.0846x$. Show that the model predicts that the number of users doubles in a fixed number of weeks, whatever the number of users at the start, and find that number of weeks. Find the week in which the model first gives $10\,000$ users. The company says that it will have 1 million users by week 52. Use the model to comment on this claim. [9 marks]

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the model as $N = kb^t$. Doubling means $kb^{t+D} = 2kb^t$: the k and the b^t cancel.

Hint 2 $b^D = 2$, so $D = \dfrac{\log 2}{\log b}$, and $\log b$ is the gradient of the coded line.

Hint 3 For $10\,000$ users, $\log N = 4$. For week 52, find N from the model, then think about how far week 52 is from the data.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can say which coding turns $y = ax^n$ or $y = kb^x$ into a straight line. ○ ○ ○

I can find a, n, k and b from the regression line of the coded data. ○ ○ ○

I can interpret the constants of a model in context, as a value or a percentage change. ○ ○ ○

I can use a model to predict, and say when a prediction is not reliable. ○ ○ ○

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