Finding μ and σ

99 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-finding-mean-and-standard-deviation/

Date

A Do Now

Quick questions to start. Some use earlier work; some set a slip that the examiner reports name.

A1 Needed today [2 marks]

Solve $\mu + 2\sigma = 50$ and $\mu - \sigma = 35$.

Answer .....

A2 Needed today [2 marks]

$Z \sim \mathrm{N}(0, 1)$. Find z such that $\mathrm{P}(Z < z) = 0.15$, to 4 decimal places.

Answer .....

A3 Common slip [2 marks]

$X \sim \mathrm{N}(\mu, 4^2)$ and $\mathrm{P}(X > 18) = 0.1$. Mo writes "$\dfrac{18 - \mu}{4} = 0.9$". Find μ correctly.

Answer .....

A4 Common slip [2 marks]

$X \sim \mathrm{N}(\mu, 9)$. Find $\mathrm{P}(X < \mu + 3)$.

Answer .....

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PART 1 Finding the mean

EXAMPLE 1 · I DO

$X \sim \mathrm{N}(\mu, 6^2)$ and $\mathrm{P}(X < 50) = 0.7$. Find μ.

Hint 50 has more than half the area to its left, so 50 is above the mean and its z value is positive.

YOUR TURN · WE DO

B1 $Y \sim \mathrm{N}(\mu, 2.5^2)$ and $\mathrm{P}(Y > 30) = 0.05$. Find μ. [3 marks]

Answer .....

PRACTICE · YOU DO

C1 A machine fills bags of flour. The masses, B grams, are modelled by $B \sim \mathrm{N}(\mu, 3.5^2)$, where the machine can be set to any mean μ. The bags are labelled 500 g. Find the least value of μ such that at most 1% of bags weigh less than 500 g. [3 marks]

Answer .....

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C2 $Y \sim \mathrm{N}(\mu, 12.25)$ and $\mathrm{P}(Y > 30) = 0.4$. Find μ, then find $\mathrm{P}(Y < 25)$. [4 marks]

Answer .....

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PART 2 Finding the standard deviation

EXAMPLE 2 · I DO

The masses, W grams, of apples are modelled by $W \sim \mathrm{N}(120, \sigma^2)$, and $\mathrm{P}(W > 130) = 0.15$. Find σ.

Hint 130 is above the mean, so its z value is positive: $\mathrm{P}(Z > 1.0364) = 0.15$.

YOUR TURN · WE DO

B2 $L \sim \mathrm{N}(40, \sigma^2)$ and $\mathrm{P}(L < 35) = 0.2$. Find σ, state $\mathrm{P}(L > 45)$ and find the variance of L. [5 marks]

Answer .....

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C3 $X \sim \mathrm{N}(40, \sigma^2)$ and $\mathrm{P}(X < 46) = 0.85$. Tom writes "$\dfrac{46 - 40}{\sigma} = 0.85$, so the variance is 7.06". Give two mistakes, and find the standard deviation and the variance. [4 marks]

C4 The times, T minutes, of a bus journey are modelled by a normal distribution with median 64. 18% of journeys take more than 70 minutes. Find the standard deviation, then find $\mathrm{P}(T < 60)$. [4 marks]

Answer .....

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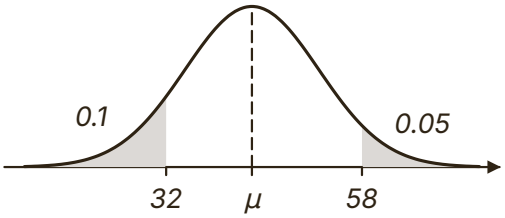
PART 3 Finding μ and σ from two probabilities

EXAMPLE 3 · I DO

$X \sim \mathrm{N}(\mu, \sigma^2)$, with $\mathrm{P}(X < 32) = 0.1$ and $\mathrm{P}(X > 58) = 0.05$, as the diagram shows. Find μ and σ.



Hint Write one equation for each value. 32 is in the lower tail, so its z value is NEGATIVE. Then subtract the equations.

YOUR TURN · WE DO

B3 $T \sim \mathrm{N}(\mu, \sigma^2)$, with $\mathrm{P}(T < 12) = 0.3$ and $\mathrm{P}(T > 20) = 0.15$. Find μ and σ. [5 marks]

Answer .....

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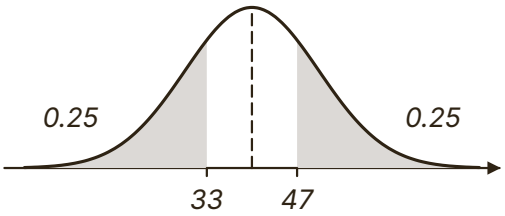
EXAMPLE 4 · I DO

$X \sim \mathrm{N}(\mu, \sigma^2)$, with $\mathrm{P}(X > 47) = 0.25$ and $\mathrm{P}(X < 33) = 0.25$. Find μ and σ.



Hint The two tails are the same size, so 33 and 47 are the same distance from the mean.

YOUR TURN · WE DO

B4 $Y \sim \mathrm{N}(\mu, \sigma^2)$, with $\mathrm{P}(Y < 16) = \mathrm{P}(Y > 28) = 0.1$. Find μ and σ, and the interquartile range of Y. [5 marks]

Answer .....

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C5 $X \sim \mathrm{N}(\mu, \sigma^2)$, with $\mathrm{P}(X < 210) = 0.9$ and $\mathrm{P}(X < 180) = 0.25$. Lily uses z values 1.28 and -0.67 and gets $\sigma = 15.38$. Find σ correctly to 4 significant figures, and μ. [4 marks]

C6 $X \sim \mathrm{N}(\mu, \sigma^2)$, with $\mathrm{P}(X < 20) = 0.1$ and $\mathrm{P}(X > 50) = 0.2$. Raj writes "$\dfrac{20 - \mu}{\sigma} = 1.2816$ and $\dfrac{50 - \mu}{\sigma} = 0.8416$" and gets $\sigma = -68.18$. Explain his mistake and find μ and σ. [5 marks]

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PART 4 Finding μ and σ, then a probability

EXAMPLE 5 · I DO

The lengths, F cm, of fish in a lake are modelled by a normal distribution. 2.5% of fish are longer than 54 cm and 2.5% are shorter than 30 cm. (i) Find the mean and show that the standard deviation is 6.12 to 3 significant figures. (ii) Twelve fish are caught. Find the probability that at least three of them are longer than 48 cm.

Hint Equal tails put the mean halfway. For a "show that", give the value to more figures than asked, then round.

YOUR TURN · WE DO

B5 The diameters, D mm, of rods are modelled by a normal distribution. 5% are less than 99.0 mm and 1% are more than 101.5 mm. Find the mean and standard deviation, then find $\mathrm{P}(99.5 < D < 100.5)$. [7 marks]

Answer .....

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C7 The lengths, L cm, of leaves on a plant are modelled by a normal distribution. 20% of the leaves are shorter than 12.0 cm and 5% are longer than 15.0 cm. Find the mean and the standard deviation of L, then find $\mathrm{P}(12.5 < L < 14.5)$. [6 marks]

Answer .....

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C8 The masses, X grams, of eggs from a farm are modelled by a normal distribution. 30% of eggs weigh more than 68 g and 9% weigh less than 50 g. Find the probability that an egg chosen at random weighs between 55 g and 65 g. [8 marks]

Answer .....

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D1

> Kim writes: "$\frac{30-\mu}{\sigma} = -1.6449$ and $\frac{70-\mu}{\sigma} = -0.8416$, so $\sigma = -49.79$."

The marks, M, in a test are modelled by a normal distribution. 5% of students score below 30 and 20% score above 70.

(a) Explain Kim's mistake. [2 marks]

(b) Find the mean and the standard deviation of M. [4 marks]

Answer .....

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(c) The pass mark is set so that 75% of students pass. Find the pass mark. [2 marks]

Answer .....

(d) Given that a student passes, find the probability that they score above 70. [2 marks]

Answer .....

Total for D1: 10 marks

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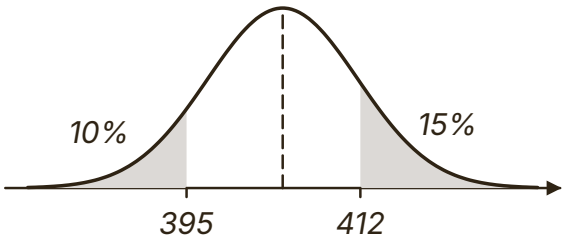
D2 The volumes, V ml, of jam in jars are modelled by $V \sim \mathrm{N}(\mu, \sigma^2)$. 10% of jars hold less than 395 ml and 15% hold more than 412 ml.

(a) Find μ and σ. [5 marks]



Answer .....

(b) Find the probability that a jar holds more than 400 ml. [2 marks]

Answer .....

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(c) Eight jars are chosen at random. Find the probability that at least six of them hold more than 400 ml. [3 marks]

Answer .....

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The lifetimes, L hours, of a battery are modelled by $L \sim \mathrm{N}(\mu, \sigma^2)$, and 15% of batteries last less than 300 hours. A new process adds 20 hours to the mean and leaves the standard deviation the same. Under the new process only 5% last less than 300 hours. (i) Find μ and σ. (ii) For a battery from the new process, find the probability that it lasts more than 400 hours, given that it lasts more than 350 hours. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write one equation for the old process and one for the new. In the new one the mean is $\mu + 20$, and the standard deviation is still σ.

Hint 2 Both z values are negative, since 300 is below the mean each time: -1.0364 for 15% and -1.6449 for 5%.

Hint 3 For (ii), "lasts more than 400" is inside "lasts more than 350", so the conditional probability is $\dfrac{\mathrm{P}(L > 400)}{\mathrm{P}(L > 350)}$, with the new mean.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find an unknown mean or standard deviation from one given probability.	○ ○ ○
I can find both μ and σ by solving two equations, with the correct sign on each z value.	○ ○ ○
I can use symmetry to find the mean straight away.	○ ○ ○
I can answer design questions, such as the least mean that keeps 1% under a limit.	○ ○ ○

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