Hypothesis testing with the normal distribution

129 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-hypothesis-testing-with-the-normal-distribution/

Date

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A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that the examiner reports say costs marks.

A1 Needed today [2 marks]

$X \sim \mathrm{N}(60, 3^2)$. Find $\mathrm{P}(X < 58.5)$.

Answer .....

A2 Needed today [2 marks]

$Z \sim \mathrm{N}(0, 1)$. Find a and b such that $\mathrm{P}(Z > a) = 0.025$ and $\mathrm{P}(Z < b) = 0.01$.

Answer .....

A3 Common slip [2 marks]

A test is about the mean volume of juice in a carton, claimed to be 250 ml. Mo writes $\mathrm{H}_0 : \bar{x} = 250$, $\mathrm{H}_1 : \bar{x} < 250$. Say what is wrong and write the hypotheses correctly.

Answer .....

A4 Last lesson [2 marks]

$X \sim \mathrm{B}(100, 0.5)$. Use a normal approximation to find $\mathrm{P}(X \geqslant 60)$.

Answer .....

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A5 Common slip [2 marks]

In a one-tailed test at the 5% level, Lee finds $z = 1.82$ and writes "$1.82 > 0.05$, so do not reject H_0". Correct his comparison and his decision.

Answer .....

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B Example, then your turn

A-Level

Every test has the same six steps. Step 1 Model: say what is measured or counted, and its distribution if H_0 is true. Step 2 Hypotheses: H_0 and H_1 in the parameter μ, never in the sample value. Step 3 Level and tails: the significance level, and one tail or two. Step 4 Calculate assuming H_0: the distribution of the sample mean is $\bar{X} \sim \mathrm{N}(\mu_0, \sigma^2/n)$, where μ_0 is the mean in H_0; find the probability of the observed mean or a more extreme one, or the critical region. Step 5 Compare: that probability with the level, or the observed value with the critical region. Step 6 Conclude: reject or do not reject H_0, then one sentence in context. The worked answers label every step.

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EXAMPLE 1

The masses, X grams, of bars of soap are modelled by $X \sim \mathrm{N}(125, 10^2)$. A random sample of 16 bars is taken.

(i) State the distribution of the sample mean $\bar{X}$.

(ii) Find $\mathrm{P}(\bar{X} > 129)$.

(iii) Compare this with the probability that one bar has a mass over 129 g.

Hint The sample mean has the same mean as X, but its variance is divided by n. Divide the variance, not the standard deviation, by n.

B1 $X \sim \mathrm{N}(42, 8^2)$. A random sample of 25 observations is taken.

(a) State the distribution of $\bar{X}$. [2 marks]

Answer .....

(b) Find $\mathrm{P}(40 < \bar{X} < 44)$. [2 marks]

Answer .....

Total for B1: 4 marks

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EXAMPLE 2

Bags of rice are labelled 1 kg. The masses, in grams, are normally distributed with mean 1000 and standard deviation 6. After a machine is reset, a random sample of 20 bags has mean mass 997.6 g. The standard deviation is unchanged. Test, at the 5% level of significance, whether the mean mass has decreased.

Hint The model in Step 1 uses the mean in H_0 (1000), not the sample mean. Find the probability of a sample mean this low or lower.

B2 The times, in hours, that a battery lasts are normally distributed with mean 24 and standard deviation 3.2. A new design is tried. A random sample of 18 batteries has mean time 25.3 hours. Assume the standard deviation is unchanged. Test, at the 5% level, whether the mean time has increased. Do the test one step at a time.

(a) Step 1 Model: state the distribution of $\bar{X}$ if H_0 is true. [1 mark]

Answer .....

(b) Step 2 Hypotheses. [1 mark]

Answer .....

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(c) Step 3 Level and tails. [1 mark]

Answer .....

(d) Step 4 Calculate assuming H_0. [2 marks]

Answer .....

(e) Step 5 Compare. [1 mark]

Answer .....

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EXAMPLE 3

Nails of a certain type have lengths, in cm, that are normally distributed with mean 8.00 and standard deviation 0.04. A new machine makes the nails. A random sample of 30 of its nails has mean length 8.016 cm. Assuming the standard deviation is unchanged, test at the 5% level whether the mean length has changed.

Hint "Changed" means two tails: 2.5% in each. Compare z with 1.96, or the probability in the tail on the side of $\bar{x}$ with 0.025.

B3 Reaction times, in seconds, are normally distributed with mean 0.42 and standard deviation 0.05. After a drink, a random sample of 25 people has mean reaction time 0.402 s. Assume the standard deviation is unchanged. Test, at the 1% level, whether the mean reaction time has changed.

(a) Steps 1 to 3: the model, the hypotheses, and the level in each tail. [3 marks]

Answer .....

(f) Step 6 Conclude, in context. [1 mark]

Total for B2: 7 marks

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(b) Steps 4 to 6. [3 marks]

EXAMPLE 4

Test scores have standard deviation 12. A test of $\mathrm{H}_0 : \mu = 65$ against $\mathrm{H}_1 : \mu \neq 65$ uses the mean of a random sample of 36 scores, at the 5% level.

(i) Find the critical region for $\bar{X}$.

(ii) The sample mean is 68.5. State the conclusion.

Hint Find the values of $\bar{X}$ that are 1.96 standard deviations of $\bar{X}$ from 65. The critical region is a set of values of $\bar{X}$, not a probability.

B4 The lifetimes of a component have standard deviation 9 hours. A test of $\mathrm{H}_0 : \mu = 500$ against $\mathrm{H}_1 : \mu < 500$ uses a random sample of 12 components, at the 1% level. Find the critical region for $\bar{X}$, and state the conclusion if the sample mean is 494.1 hours. [5 marks]

Total for B3: 6 marks

Answer .....

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EXAMPLE 5

The lengths, in mm, of a species of beetle are normally distributed with standard deviation 2.5. The mean used to be 50. A random sample of 8 beetles from a new site has lengths: 51.2, 49.8, 52.4, 50.9, 51.7, 50.3, 52.0, 51.1

(i) Test, at the 5% level, whether the mean length at the new site is greater than 50 mm.

(ii) Find the least sample size for which a sample mean of 50.6 mm would lead to rejecting H_0.

Hint Find $\bar{x}$ first. For (ii), the sample mean must be at least 1.6449 standard deviations of $\bar{X}$ above 50; that standard deviation is $\frac{2.5}{\sqrt{n}}$.

B5 [7 marks]

A machine fills bags of sugar. The masses, in grams, are normally distributed with standard deviation 4, and the mean should be 500. A random sample of 10 bags has masses: 497.2, 503.1, 495.8, 498.4, 496.9, 499.5, 494.3, 501.0, 497.6, 495.2

Test, at the 5% level of significance, whether the mean mass is different from 500 g.

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Answer .....

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C1 The strength of a type of concrete, in suitable units, is normally distributed with mean 32 and standard deviation 2.4. A new mix is tried. A random sample of 15 blocks has mean strength 33.7. Assume the standard deviation is unchanged. Test, at the 1% level, whether the mean strength has increased. Do one step in each part.

(a) Step 1 Model and Step 2 Hypotheses. [2 marks]

Answer .....

(b) Step 3 Level and tails, and Step 4 Calculate assuming H_0. [2 marks]

Answer .....

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(c) Step 5 Compare and Step 6 Conclude. [2 marks]

Total for C1: 6 marks

C2 Cartons of juice hold volumes, in ml, that are normally distributed with standard deviation 4. A test of $\mathrm{H}_0 : \mu = 250$ against $\mathrm{H}_1 : \mu < 250$ is done at the 5% level. A random sample of 16 cartons has mean 248.1 ml. Amir writes "$\mathrm{P}\left(Z < \frac{248.1-250}{4}\right) = 0.3174$, so do not reject H_0." Explain his error and complete the test. [4 marks]

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C3 The wing lengths, in mm, of a type of bird are normally distributed with mean 46 and standard deviation 3.1. A random sample of 24 birds from an island has mean wing length 47.2 mm. Bea tests whether the mean wing length on the island is greater than 46 mm, at the 5% level. She writes: "$\mathrm{H}_0 : \mu = 47.2$, $\mathrm{H}_1 : \mu > 47.2$. $z = \frac{47.2-46}{3.1/\sqrt{24}} = 1.8964$. $1.8964 > 0.05$, so do not reject H_0."

(a) Describe her two errors. [2 marks]

(b) Complete the test correctly. [4 marks]

Total for C3: 6 marks

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C4 Chocolate bars have masses, in grams, that are normally distributed with standard deviation 1.5. The mean should be 52. A random sample of 20 bars is used to test whether the mean has changed, at the 5% level. Sian writes "Critical region: $\mathrm{P}(\bar{X} \geqslant 52.55) = 0.05$." Explain her two mistakes and find the correct critical region. [4 marks]

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C5 Resting pulse rates of adults, in beats per minute, are normally distributed with mean 72 and standard deviation 7.5. A random sample of 30 adults who swim each day has mean pulse rate 69.9. Assuming the standard deviation is the same for swimmers, test at the 10% level whether their mean pulse rate is different from 72. [6 marks]

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D Exam-style questions

Edexcel exam style

D1 The masses, M kg, of adult dogs of one breed are modelled by $M \sim \mathrm{N}(28, 3.2^2)$.

(a) Find the probability that one dog chosen at random has a mass over 30 kg. [2 marks]

Answer .....

(b) A random sample of 16 dogs is taken. Find the probability that their mean mass is over 29 kg. [3 marks]

Answer .....

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(c) A vet thinks dogs of this breed on a new diet are lighter. A random sample of 16 dogs on the diet has mean mass 26.7 kg. Test the vet's belief at the 5% level, assuming the standard deviation is unchanged. [5 marks]

Total for D1: 10 marks

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D2

The lengths of a type of seed are normally distributed with mean 31 mm and standard deviation 2.4 mm. A gardener wants to know whether seeds from a new supplier have a different mean length. A random sample of 36 of these seeds has mean length 31.75 mm. Kim writes: "$\mathrm{H}_0 : \bar{x} = 31$, $\mathrm{H}_1 : \bar{x} > 31$. $\bar{X} \sim \mathrm{N}(31.75, 2.4^2)$. $\mathrm{P}(\bar{X} > 31) = 0.6227$. $0.6227 > 0.05$, so accept H_0. This proves the mean length is still 31 mm."

Kim's working has several errors.

(a) Describe four errors in Kim's working. [4 marks]

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(b) Carry out the test correctly at the 5% level. [6 marks]

Answer .....

Total for D2: 10 marks

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D3 A machine makes metal discs. Their diameters, in mm, are normally distributed with mean 12.00 and standard deviation 0.06. After the machine is moved, a random sample of 40 discs has mean diameter 11.978 mm. Assuming the standard deviation is unchanged, test at the 1% level of significance whether the mean diameter has decreased. [7 marks]

Answer .....

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D4 The volumes, in ml, of drinks in cans from a factory are normally distributed with standard deviation 2.5. The mean should be 330. Each day a random sample of 25 cans is taken to test $\mathrm{H}_0 : \mu = 330$ against $\mathrm{H}_1 : \mu \neq 330$ at the 1% level.

(a) Find the critical region for $\bar{X}$. [4 marks]

Answer .....

(b) On one day the sample mean is 331.4 ml. State the conclusion. [2 marks]

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(c) Find the p-value for this sample, and state the smallest significance level at which H_0 would be rejected. [2 marks]

Answer .....

(d) State two assumptions needed for the test to be valid. [2 marks]

Total for D4: 10 marks

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D5 The reaction times, in seconds, of drivers in a simulator are normally distributed with mean 3.2 and standard deviation 0.8. An instructor thinks that drivers who have had extra training react faster.

(a) A random sample of 10 trained drivers has mean reaction time 2.85 s. Test the instructor's belief at the 5% level, assuming the standard deviation is unchanged. [5 marks]

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(b) Find the least number of trained drivers in a sample for which a mean reaction time of 2.95 s would lead to the instructor's belief being supported at the 5% level. [4 marks]

Answer .....

Total for D5: 9 marks

D6 The masses, in kg, of adult rabbits in a large colony are normally distributed with mean 2.40. Of the rabbits, 2.5% have a mass greater than 2.80 kg.

(a) Show that the standard deviation of the masses is 0.204 kg to 3 significant figures. [3 marks]

Answer .....

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(b) After a dry summer, a random sample of 15 rabbits from the colony has mean mass 2.31 kg. Assuming the standard deviation is unchanged, test at the 5% level whether the mean mass has changed. [6 marks]

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Exam marks are normally distributed with standard deviation 6. A teacher tests $\mathrm{H}_0 : \mu = 80$ against $\mathrm{H}_1 : \mu > 80$, using the mean of a random sample of n marks. The sample mean is 81.5. Find the set of values of n for which the result is significant at the 5% level but not at the 1% level. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write z in terms of n. It simplifies.

Hint 2 Significant at 5% means $z > 1.6449$; not significant at 1% means $z \leqslant 2.3263$.

Hint 3 Square each inequality, then take whole numbers. Check the end values.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can state the distribution of the sample mean, N(μ, σ²/n), and find probabilities for it.	○ ○ ○
I can test a population mean using the six steps, one-tailed or two-tailed, by probability or by z.	○ ○ ○
I can find a critical region for the sample mean and use it to reach a conclusion in context.	○ ○ ○
I can find the smallest sample size that makes a result significant.	○ ○ ○

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