The inverse normal and the standard normal

95 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-inverse-normal/

Date

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A Do Now

Quick questions to start. Some use earlier work; some set a slip that the examiner reports name.

A1 Needed today [2 marks]

$X \sim \mathrm{N}(70, 8^2)$. Find $\mathrm{P}(X > 82)$.

Answer .....

A2 Needed today [2 marks]

A data set x has mean 70 and standard deviation 8. It is coded by $y = \dfrac{x - 70}{8}$. Find the mean and the standard deviation of y.

Answer .....

A3 Common slip [2 marks]

A calculator's inverse normal function gives the value a with $\mathrm{P}(X < a)$ equal to the area you enter. To find b with $\mathrm{P}(X > b) = 0.15$, what area do you enter? Is b above or below the mean?

Answer .....

A4 Common slip [2 marks]

$X \sim \mathrm{N}(30, 16)$. Find the z value for $x = 36$.

Answer .....

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A5 Common slip [2 marks]

For a distribution the 10 th percentile is 21.4 and the 90th percentile is 37.8. State the 10% to 90% interpercentile range.

Answer .....

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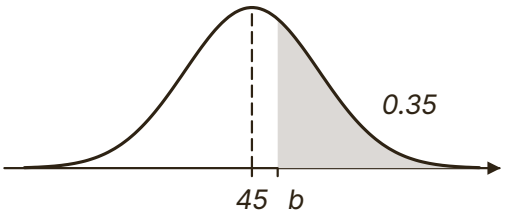
PART 1 Finding a value from a probability

EXAMPLE 1 · I DO

$X \sim \mathrm{N}(45, 6^2)$. Find (i) a such that $\mathrm{P}(X < a) = 0.8$ (ii) b such that $\mathrm{P}(X > b) = 0.35$, the shaded area (iii) c such that $\mathrm{P}(c < X < 45) = 0.3$.



Hint The calculator works with the area to the LEFT. For (ii) the area to the left of b is 0.65. For (iii) half the area is below 45 .

YOUR TURN · WE DO

B1 $Y \sim \mathrm{N}(120, 15^2)$. Find (i) a such that $\mathrm{P}(Y < a) = 0.9$ (ii) b such that $\mathrm{P}(Y > b) = 0.05$ (iii) d such that $\mathrm{P}(110 < Y < d) = 0.4$. [5 marks]

Answer .....

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C1 $X \sim \mathrm{N}(50, 4^2)$. Find (i) a such that $\mathrm{P}(X < a) = 0.75$ (ii) b such that $\mathrm{P}(X > b) = 0.2$. [3 marks]

Answer .....

C2 $X \sim \mathrm{N}(60, 8^2)$ and $\mathrm{P}(X > a) = 0.1$. Dev writes "$\dfrac{a - 60}{8} = 0.1$, so $a = 60.8$". Explain his mistake and find a. [4 marks]

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C3 $X \sim \mathrm{N}(24, 3^2)$. Find a such that $\mathrm{P}(20 < X < a) = 0.5$. [4 marks]

Answer .....

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PART 2 Quartiles and interpercentile ranges

EXAMPLE 2 · I DO

The diameters, D mm, of washers are modelled by $D \sim \mathrm{N}(80, 5^2)$. Find (i) the interquartile range (ii) the 5% to 95% interpercentile range (iii) k such that $\mathrm{P}(80 - k < D < 80 + k) = 0.75$.

Hint The lower quartile has 25% of the area to its left. Keep full calculator values and round only the final answer.

YOUR TURN · WE DO

B2 The masses, M grams, of chocolate bars are modelled by $M \sim \mathrm{N}(250, 12^2)$. Find (i) the interquartile range (ii) the 10% to 90% interpercentile range (iii) k such that $\mathrm{P}(250 - k < M < 250 + k) = 0.6$. [6 marks]

Answer .....

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C4 $X \sim \mathrm{N}(37.5, 6.3^2)$. Sam is asked for the 10% to 90% interpercentile range. He writes "10th percentile 29.4, 90th percentile 45.6, so the range is $29.4 < X < 45.6$". Explain what is wrong and find the interpercentile range to 3 significant figures. [4 marks]

C5 The masses, W grams, of packs of cheese are modelled by $W \sim \mathrm{N}(320, 6^2)$. An outlier is a value below $Q_1 - 1.5 \times \mathrm{IQR}$ or above $Q_3 + 1.5 \times \mathrm{IQR}$. Find Q_1, Q_3 and the two outlier limits. [4 marks]

Answer .....

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PART 3 Standardising with Z

EXAMPLE 3 · I DO

$X \sim \mathrm{N}(72, 5^2)$ and $Z \sim \mathrm{N}(0, 1)$, with $\Phi(z) = \mathrm{P}(Z < z)$. (i) Write $\mathrm{P}(X < 78)$, $\mathrm{P}(X \geqslant 64)$ and $\mathrm{P}(66 < X < 80)$ in terms of Φ, and evaluate them. (ii) Use the percentage points table, $\mathrm{P}(Z > 1.6449) = 0.05$, to find the value of X exceeded by 5% of the distribution. Solutions relying entirely on calculator technology are not acceptable.

Hint $Z = \dfrac{X - \mu}{\sigma}$ codes X so that it has mean 0 and standard deviation 1. By symmetry $\Phi(-z) = 1 - \Phi(z)$.

YOUR TURN · WE DO

B3 $X \sim \mathrm{N}(140, 20^2)$. (i) Write $\mathrm{P}(X < 165)$ and $\mathrm{P}(X > 118)$ in terms of Φ, and evaluate them. (ii) Given $\mathrm{P}(Z > 2.3263) = 0.01$, find the value of X exceeded by 1% of the distribution, showing the standardisation. [6 marks]

Answer .....

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EXAMPLE 4 · I DO

The scores, S, in an aptitude test are modelled by $S \sim \mathrm{N}(64, 9^2)$. A company interviews the people with the highest 15% of scores and offers extra training to the lowest 10%. Find the score needed for an interview and the score below which training is offered. Solutions relying entirely on calculator technology are not acceptable.

Hint Sketch both tails. The upper tail has area 0.15: $\mathrm{P}(Z > 1.0364) = 0.15$. The lower tail of 0.10 has a NEGATIVE z value.

YOUR TURN · WE DO

B4 The heights, H cm, of adult men are modelled by $H \sim \mathrm{N}(176, 7.5^2)$. (i) A door is to be made so that only 0.5% of men are taller than it. Find the height of the door, using $\mathrm{P}(Z > 2.5758) = 0.005$. (ii) A seat is designed for men above the 5th percentile. Find the 5 th percentile, showing the standardisation. [4 marks]

Answer .....

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C6 $X \sim \mathrm{N}(200, 15^2)$ and $\mathrm{P}(X < a) = 0.1$. Ella writes "$\dfrac{a - 200}{15} = 1.28$, so $a = 219.2$". Give two mistakes and find a. [4 marks]

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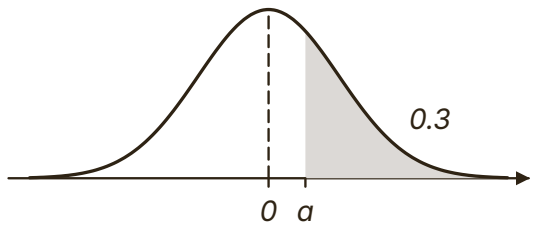
C7 $Z \sim \mathrm{N}(0, 1)$ and $\mathrm{P}(Z > a) = 0.3$, as shaded in the diagram. (i) Without a calculator, find $\mathrm{P}(-a < Z < a)$ and $\mathrm{P}(Z < -a \mid Z < a)$. (ii) Find a. [4 marks]



Answer .....

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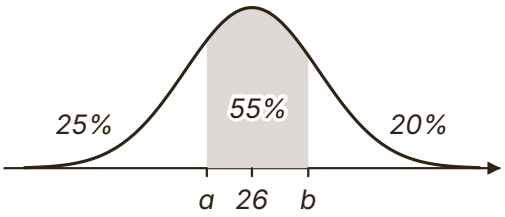
PART 4 Values from a conditional probability

EXAMPLE 5 · I DO

Foot lengths, F cm, of a shop's customers are modelled by $F \sim \mathrm{N}(26.0, 1.2^2)$. The shop stocks small, medium and large boots in the proportions 25%, 55% and 20%, as the diagram shows. (i) Find the foot lengths at which medium starts and ends. (ii) State one assumption you have made.



Hint Small is the lowest 25%, so the start of medium, a, has 0.25 to its left. Large is the highest 20%, so b has 0.8 to its left.

YOUR TURN · WE DO

B5 $T \sim \mathrm{N}(30, 4^2)$. (i) Find t such that $\mathrm{P}(T > t \mid T > 30) = 0.2$. (ii) Find s such that $\mathrm{P}(T < s \mid T < 30) = 0.5$. [4 marks]

Answer .....

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C8 $T \sim \mathrm{N}(50, 10^2)$. Find t such that $\mathrm{P}(T > t \mid T > 50) = 0.3$. [3 marks]

Answer .....

C9 $M \sim \mathrm{N}(70, 5^2)$. Find m such that $\mathrm{P}(M < m \mid M > 65) = 0.5$. [4 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1

> Jay says: "$\mathrm{P}(T < t) = 0.95$, so $z = 1.96$ and $t = 38 + 1.96 \times 6 = 49.76$."

The times, T minutes, for a firm to deliver a meal are modelled by $T \sim \mathrm{N}(38, 6^2)$. The firm wants to promise that 95% of meals arrive within t minutes.

(a) Explain Jay's mistake. [2 marks]

(b) Find t. [2 marks]

Answer .....

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(c) Find the interval, symmetric about the mean, that holds the middle 95% of delivery times. [3 marks]

Answer .....

(d) Given that a delivery takes longer than the mean, find the probability that it takes longer than 50 minutes. [3 marks]

Answer .....

Total for D1: 10 marks

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D2 The marks, M, in an exam are modelled by $M \sim \mathrm{N}(58, 12^2)$. The highest 10% of marks get grade A and the next 25% get grade B.

(a) Find the lowest mark for grade A and the lowest mark for grade B. Show the standardisation. [4 marks]

Answer .....

(b) Find the percentage of students with a mark above 70. [2 marks]

Answer .....

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(c) Given that a student got grade B or better, find the probability that they got grade A. [2 marks]

Answer .....

Total for D2: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $X \sim \mathrm{N}(100, 15^2)$. (i) Find the value of k such that $\mathrm{P}(X > 100 + k \mid X > 100 - k) = 0.1$. (ii) Two independent observations of X are taken, and both are greater than $100 - k$. Find the probability that at least one of them is greater than $100 + k$. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Call $\mathrm{P}(X > 100 + k) = p$. The two values $100 + k$ and $100 - k$ are the same distance from the mean. What is $\mathrm{P}(X > 100 - k)$ in terms of p?

Hint 2 The event $X > 100 + k$ is inside the event $X > 100 - k$, so the conditional probability is $\dfrac{p}{1 - p}$. Solve $\dfrac{p}{1 - p} = 0.1$.

Hint 3 Once you have p, use the inverse normal to find $100 + k$. For (ii), each observation is now a trial with probability 0.1.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find a when $\mathrm{P}(X < a)$, $\mathrm{P}(X > a)$ or $\mathrm{P}(c < X < a)$ is given. ○ ○ ○

I can find quartiles, the interquartile range and an interpercentile range for a normal distribution. ○ ○ ○

I can standardise with $Z = \dfrac{X - \mu}{\sigma}$, write a probability as $\Phi(z)$, and use z values to 4 decimal places. ○ ○ ○

I can find boundaries and thresholds in context and state an assumption. ○ ○ ○

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