The normal distribution and its probabilities

95 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-normal-distribution/

Date

A Do Now

Quick questions to start. Some use earlier work; some set a slip that the examiner reports name.

A1 Needed today [2 marks]

A data set has mean 12 and variance 6.25. State the standard deviation, and find the value that is 2 standard deviations above the mean.

Answer .....

A2 Needed today [2 marks]

$X \sim \mathrm{B}(10, 0.2)$. Find $\mathrm{P}(X \geqslant 3)$.

Answer .....

A3 Needed today [2 marks]

$\mathrm{P}(A) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$. Find $\mathrm{P}(B \mid A)$.

Answer .....

A4 Common slip [2 marks]

X is a continuous random variable. Kim says $\mathrm{P}(X < 20)$ and $\mathrm{P}(X \leqslant 20)$ are different because one of them includes 20. Explain why they are equal.

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PART 1 The shape of a normal curve

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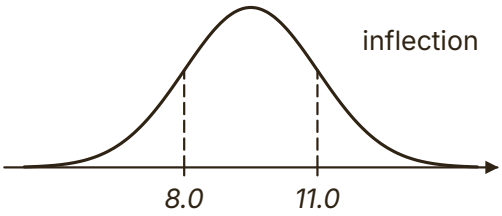
EXAMPLE 1 · I DO

The lengths, L cm, of the leaves on a tree are modelled by a normal distribution. The diagram shows the curve. Its points of inflection are at $L = 8.0$ and $L = 11.0$. (i) State the mean and the standard deviation of L. (ii) Without a calculator, estimate the percentage of leaves longer than 12.5 cm.



Hint The points of inflection are one standard deviation either side of the mean. About 95% of values lie within 2 standard deviations of the mean.

YOUR TURN · WE DO

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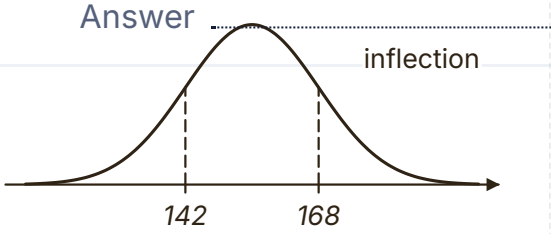
PRACTICE · YOU DO

C1 The heights, H cm, of a type of plant are modelled by a normal distribution. The curve has its points of inflection at $H = 21$ and $H = 27$. (i) State the mean and the standard deviation of H. (ii) Without a calculator, estimate the percentage of plants with heights between 21 cm and 30 cm. [3 marks]

B1 The masses, M grams, of apples from an orchard are modelled by a normal distribution. The diagram shows the curve, with its points of inflection at $M = 142$ and $M = 168$. (i) State the mean and the standard deviation of M. (ii) Without a calculator, estimate the percentage of apples with mass between 129 g and 168 g. (iii) Estimate $\mathrm{P}(M < 116)$. [5 marks]

Answer



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C2 The lengths of 200 bolts are measured. The distribution is roughly symmetric. 141 of the lengths are within one standard deviation of the mean and 189 are within two standard deviations of the mean. Lou says this rules out a normal model because 141 is not 68% of 200. Comment on Lou's claim.

Answer ..... [4 marks]

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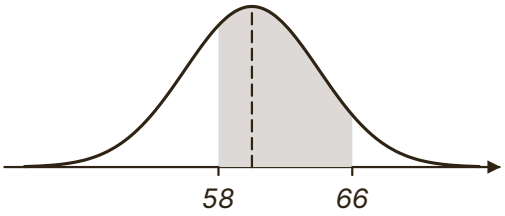
PART 2 Normal probabilities on a calculator

EXAMPLE 2 · I DO

$X \sim \mathrm{N}(60, 16)$. Find (i) $\mathrm{P}(X > 65)$ (ii) $\mathrm{P}(X \leqslant 57)$ (iii) $\mathrm{P}(58 < X < 66)$ (iv) $\mathrm{P}(X < 55 \text{ or } X > 67)$.



Hint The second number in $\mathrm{N}(60, 16)$ is the variance. The calculator needs the standard deviation. Sketch each region first: is the answer more or less than 0.5?

YOUR TURN · WE DO

B2 $Y \sim \mathrm{N}(25, 6.25)$. Find [6 marks] (i) $\mathrm{P}(Y < 22)$ (ii) $\mathrm{P}(Y \geqslant 28.5)$ (iii) $\mathrm{P}(21 < Y < 27)$. (iv) State $\mathrm{P}(Y = 25)$.

Answer .....

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C3 $X \sim \mathrm{N}(30, 25)$. Ana enters $\mu = 30$ and $\sigma = 25$ in her calculator and finds $\mathrm{P}(X > 35) = 0.4207$. Explain her mistake and find $\mathrm{P}(X > 35)$. [4 marks]

C4 $W \sim \mathrm{N}(18, 2^2)$. Ben writes "$\mathrm{P}(W < 15) = 1 - 0.0668 = 0.9332$". Explain why his answer cannot be right without doing any calculation, and give the correct value. Then find $\mathrm{P}(15 < W < 20)$. [4 marks]

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C5 The time, T seconds, to complete a task is modelled by $T \sim \mathrm{N}(310, 25^2)$. Cara wants the probability that the task takes less than 5 minutes. She writes "$\mathrm{P}(T < 299.5) = 0.3372$". Explain her mistake and find the correct probability. [4 marks]

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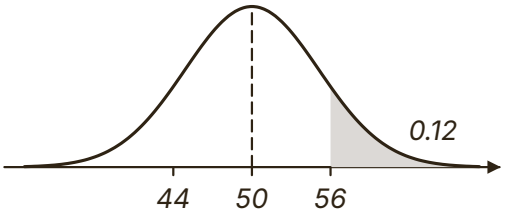
PART 3 Using the symmetry of the curve

EXAMPLE 3 · I DO

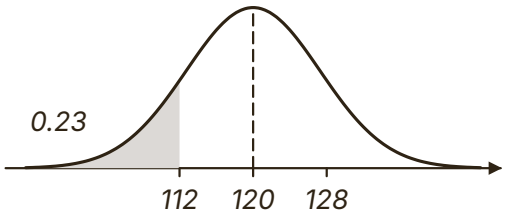
$X \sim \mathrm{N}(50, \sigma^2)$ and $\mathrm{P}(X > 56) = 0.12$, as shaded in the diagram. Without using a calculator, find (i) $\mathrm{P}(X < 44)$ (ii) $\mathrm{P}(44 < X < 56)$ (iii) $\mathrm{P}(50 < X < 56)$ (iv) $\mathrm{P}(X < 56 \mid X > 50)$.



Hint 44 and 56 are the same distance from the mean. The curve is symmetric about 50, and half the area is on each side of it.

YOUR TURN · WE DO

B3 $T \sim \mathrm{N}(120, \sigma^2)$ and $\mathrm{P}(T < 112) = 0.23$, as shaded in the diagram. Without using a calculator, find (i) $\mathrm{P}(T > 128)$ (ii) $\mathrm{P}(112 < T < 128)$ (iii) $\mathrm{P}(T > 112 \mid T < 128)$. [5 marks]



Answer .....

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C6 $X \sim \mathrm{N}(80, \sigma^2)$ and $\mathrm{P}(X > 86) = 0.15$. Without using a calculator, find (i) $\mathrm{P}(X < 74)$ (ii) $\mathrm{P}(74 < X < 86)$. [3 marks]

Answer .....

C7 $Y \sim \mathrm{N}(35, \sigma^2)$ and $\mathrm{P}(Y < 31) = 0.28$. Without using a calculator, find (i) $\mathrm{P}(35 < Y < 39)$ (ii) $\mathrm{P}(Y < 39 \mid Y > 31)$. [4 marks]

Answer .....

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PART 4 Conditional probability

EXAMPLE 4 · I DO

The time, T minutes, of a commuter's train journey is modelled by $T \sim \mathrm{N}(42, 3.5^2)$. (i) Find the probability that a journey takes more than 45 minutes. (ii) The commuter makes 5 journeys, independently. Find the probability that all 5 take more than 45 minutes, and the probability that exactly 2 of them do. (iii) Find the expected number of journeys out of 200 that take more than 45 minutes.

Hint Find $p = \mathrm{P}(T > 45)$ first. Each journey is then a trial with probability p: define a new binomial random variable.

YOUR TURN · WE DO

B4 Bags of flour have masses, F grams, with $F \sim \mathrm{N}(1010, 6^2)$. The label says 1000 g. (i) Find the probability that a bag has mass less than 1000 g. (ii) A shop buys 12 bags. Find the probability that at least one of them is under 1000 g. (iii) Find the expected number of bags under 1000 g in a delivery of 500 bags. [5 marks]

Answer .....

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EXAMPLE 5 · I DO

A runner's times, T seconds, for the 100 m are modelled by $T \sim \mathrm{N}(13.2, 0.4^2)$. (i) Given that a time is under 13.5 seconds, find the probability that it is under 13 seconds. (ii) The model gives a probability to negative times, which cannot happen. Explain why the model can still be suitable.

Hint In (i), the event "under 13" sits inside the event "under 13.5", so the intersection is just $T < 13$. In (ii), find how far 0 is from the mean in standard deviations.

YOUR TURN · WE DO

B5 The length of a phone call, C minutes, is modelled by $C \sim \mathrm{N}(6.5, 2.2^2)$. (i) Find the probability that a call lasts less than 4 minutes 30 seconds. (ii) Given that a call lasts more than 5 minutes, find the probability that it lasts more than 8 minutes. [5 marks]

Answer .....

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C8 The times, T minutes, that students take to finish a puzzle are modelled by $T \sim \mathrm{N}(14, 3.5^2)$. Given that a student takes more than 12 minutes, find the probability that they take less than 16 minutes. [3 marks]

Answer .....

C9 A sample of 80 journeys to school has distances with mean 6.1 km and standard deviation 5.3 km. The shortest distance is 0.4 km. Kai says the distance D km can be modelled by $D \sim \mathrm{N}(6.1, 5.3^2)$. Find $\mathrm{P}(D < 0)$ under Kai's model and use it to comment on his model. [4 marks]

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D1

> Lena says: "Medium is $\mathrm{P}(E < 63) - \mathrm{P}(E < 53) = 0.5433$, so large is $1 - 0.5433 = 0.4567$."

The masses, E grams, of eggs from a farm are modelled by $E \sim \mathrm{N}(62, 5^2)$. Eggs are graded: small under 53 g, medium from 53 g to 63 g, large from 63 g to 73 g, very large over 73 g.

(a) Find the proportion of eggs in each of the four grades. [4 marks]

Answer .....

(b) Explain the mistake in Lena's working. [2 marks]

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(c) The farm sells small eggs for 15p, medium for 20p, large for 25 p and very large for 30p. Find the expected income from 1000 eggs. [3 marks]

Answer .....

Total for D1: 9 marks

D2 A machine fills cups with drink. The volume, V ml, in a cup is modelled by $V \sim \mathrm{N}(250, 4^2)$. A cup overflows if $V > 258$.

(a) Find $\mathrm{P}(V < 245)$. [2 marks]

Answer .....

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(b) Find the probability that a cup overflows. [2 marks]

Answer .....

(c) 20 cups are filled. Find the probability that at least 2 of them overflow. [3 marks]

Answer .....

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(d) Given that a cup does not overflow, find the probability that it holds more than 250 ml. [3 marks]

Answer .....

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 [9 marks]

Rods are made by two machines. The lengths, in cm, of machine A's rods are modelled by $\mathrm{N}(80, 0.5^2)$ and those of machine B's rods by $\mathrm{N}(80.4, 0.8^2)$. A rod is acceptable if its length is between 79.2 cm and 80.8 cm; otherwise it is rejected. A fraction k of the rods is made by machine A and the rest by machine B. A rejected rod is equally likely to have come from either machine. (i) Find k. (ii) Rods are packed at random in boxes of 10. A box is accepted if at most one of its rods is rejected. Find the probability that a box is accepted.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Find the probability that a rod is rejected for each machine separately.

Hint 2 "Rejected and made by A" is $k \times \mathrm{P}(\text{an A rod is rejected})$. "Equally likely" means this equals "rejected and made by B".

Hint 3 Find the total probability that a rod is rejected. The number rejected in a box of 10 is binomial.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can state the properties of $X \sim \mathrm{N}(\mu, \sigma^2)$ and read μ and σ from a sketch. ○ ○ ○

I can find $\mathrm{P}(X < a)$, $\mathrm{P}(X > a)$ and $\mathrm{P}(a < X < b)$ on a calculator, with a sketch, using the standard deviation. ○ ○ ○

I can use symmetry, and use a normal probability inside a binomial or conditional probability question. ○ ○ ○

I can say, with a reason in context, whether a normal model is suitable. ○ ○ ○

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