Probability formulae

99 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-probability-formulae/

Date

A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks in the exam.

A1 Needed today [2 marks]

$X \sim \mathrm{B}(5, 0.3)$. Find $\mathrm{P}(X = 2)$.

Answer .....

A2 Needed today [3 marks]

The random variable X has $\mathrm{P}(X = x) = kx$ for $x = 1, 2, 3, 4$. Find k and $\mathrm{P}(X \geqslant 3)$.

Answer .....

A3 Needed today [2 marks]

$\mathrm{P}(A) = 0.5$ and $\mathrm{P}(B \mid A) = 0.4$. Find $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A \cap B')$.

Answer .....

A4 Common slip [2 marks]

$\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$. Find $\mathrm{P}(A \cup B)$ and $\mathrm{P}((A \cup B)')$.

Answer .....

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PART 1 The addition and multiplication formulae

EXAMPLE 1 · I DO

$\mathrm{P}(A) = 0.55$, $\mathrm{P}(B) = 0.35$ and $\mathrm{P}(A' \cap B') = 0.25$. Find (a) $\mathrm{P}(A \cup B)$, (b) $\mathrm{P}(A \cap B)$, (c) $\mathrm{P}(A \cap B')$, (d) $\mathrm{P}(A \mid B')$.

Hint The addition formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ links four numbers. Know three, find the fourth.

YOUR TURN · WE DO

B1 $\mathrm{P}(A') = 0.62$, $\mathrm{P}(B) = 0.45$ and $\mathrm{P}(A \cap B') = 0.2$.

(a) Find $\mathrm{P}(A \cap B)$. [2 marks]

Answer .....

(b) Find $\mathrm{P}(A \cup B)$. [2 marks]

Answer .....

(c) Find $\mathrm{P}(B \mid A)$. [2 marks]

Answer .....

Total for B1: 6 marks

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EXAMPLE 2 · I DO

$\mathrm{P}(A) = 0.3$, $\mathrm{P}(B \mid A) = 0.6$ and $\mathrm{P}(B \mid A') = 0.2$. Find (a) $\mathrm{P}(A \cap B)$, (b) $\mathrm{P}(B)$, (c) $\mathrm{P}(A \mid B)$.

Hint $\mathrm{P}(A \cap B) = \mathrm{P}(B \mid A) \times \mathrm{P}(A)$. B happens either with A or with A', so add those two.

YOUR TURN · WE DO

B2 25% of a company's orders come from abroad. 36% of orders from abroad arrive late, and 12% of the other orders arrive late. An order is chosen at random. F is the event "the order comes from abroad" and L is the event "the order arrives late".

(a) Find $\mathrm{P}(F \cap L)$ and $\mathrm{P}(L)$. [3 marks]

Answer .....

(b) Find $\mathrm{P}(F \mid L)$. [2 marks]

Answer .....

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(c) Find $\mathrm{P}(F' \mid L')$. [2 marks]

PRACTICE · YOU DO

C1 For two events, $\mathrm{P}(A) = 0.4$, $\mathrm{P}(B) = 0.3$ and $\mathrm{P}(A \mid B) = 0.5$. Ben writes: "$\mathrm{P}(A \cap B) = 0.4 \times 0.3 = 0.12$." Explain Ben's mistake. Find $\mathrm{P}(A \cap B)$, $\mathrm{P}(A \cup B)$ and $\mathrm{P}(B \mid A)$. [5 marks]

Answer .....

Total for B2: 7 marks

Answer .....

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C2 For two events, $\mathrm{P}(A) = 0.2$, $\mathrm{P}(B \mid A) = 0.7$ and $\mathrm{P}(B) = 0.35$. Raj writes: "$\mathrm{P}(A \mid B) = \mathrm{P}(B \mid A) = 0.7$." Explain Raj's mistake. Find $\mathrm{P}(A \mid B)$ and $\mathrm{P}(A \mid B')$. [5 marks]

Answer .....

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PART 2 Conditional probability for a random variable

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EXAMPLE 3 · I DO

$X \sim \mathrm{B}(8, 0.25)$. Find (a) $\mathrm{P}(X = 2 \mid X \geqslant 1)$, (b) $\mathrm{P}(X \leqslant 3 \mid X \geqslant 2)$. Give your answers to 3 significant figures.

Hint Work out the "given" probability first, using $1 - \mathrm{P}(X = 0)$ or $1 - \mathrm{P}(X \leqslant 1)$. Then find the values of X in both events.

YOUR TURN · WE DO

B3 Each seed in a packet germinates with probability 0.4, independently. Ten seeds are planted and X is the number that germinate.

(a) Find $\mathrm{P}(X \geqslant 4)$. [2 marks]

Answer .....

(b) Given that at least 4 seeds germinate, find the probability that exactly 5 germinate. [2 marks]

Answer .....

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(c) Given that at least 4 seeds germinate, find the probability that at least 6 germinate. [2 marks]

PRACTICE · YOU DO

C3 The discrete random variable X has the distribution shown. Kofi writes: "$\mathrm{P}(X \leqslant 3 \mid X \geqslant 2) = \dfrac{\mathrm{P}(X \leqslant 3)}{\mathrm{P}(X \geqslant 2)} = \dfrac{0.85}{0.65}$." Explain how you can tell at once that Kofi is wrong, and identify his mistake. Find the correct value. [5 marks]

Answer .....

Total for B3: 6 marks

x	0	1	2	3	4
$\mathrm{P}(X = x)$	0.15	0.2	0.25	0.25	0.15

Answer .....

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C4 The random variable X has $\mathrm{P}(X = x) = kx^2$ for $x = 1, 2, 3, 4$. [8 marks]
Two independent values X_1 and X_2 of X are taken. Find the value of k, and find the probability that $X_1 = 4$, given that $X_1 + X_2 \geqslant 7$.

Answer .....

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PART 3 Proving results about independence

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EXAMPLE 4 · I DO

A and B are independent events with $0 < \mathrm{P}(B) < 1$. (a) Show that A and B' are independent. (b) Hence show that $\mathrm{P}(A \mid B') = \mathrm{P}(A)$.

Hint A splits into $A \cap B$ and $A \cap B'$, so $\mathrm{P}(A \cap B') = \mathrm{P}(A) - \mathrm{P}(A \cap B)$. Then use independence.

YOUR TURN · WE DO

B4 A and B are events with $\mathrm{P}(A) > 0$ and $\mathrm{P}(B) > 0$.

(a) Show that if $\mathrm{P}(A \mid B) = \mathrm{P}(A)$ then $\mathrm{P}(B \mid A) = \mathrm{P}(B)$. [3 marks]

(b) Show that if $\mathrm{P}(A \mid B) > \mathrm{P}(A)$ then $\mathrm{P}(B \mid A) > \mathrm{P}(B)$. [3 marks]

Total for B4: 6 marks

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C5 $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.5$ and $\mathrm{P}(A \cap B) = r$. A and B are not independent, and $\mathrm{P}(A \mid B) \times \mathrm{P}(B \mid A) = \mathrm{P}(A \cap B)$. Kim writes: "$\dfrac{r}{0.5} \times \dfrac{r}{0.3} = r$. Dividing by r, $r = 0.15$." Explain why Kim's answer cannot be right, and what she missed. Find r and $\mathrm{P}(A \cup B)$. [6 marks]

Answer .....

C6 A and B are independent events.

(a) Show that A' and B' are independent. [3 marks]

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(b) Given that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(A' \cap B') = 0.42$, find $\mathrm{P}(B)$. [3 marks]

Answer .....

(c) Find $\mathrm{P}(A \mid A \cup B)$. [3 marks]

Answer .....

Total for C6: 9 marks

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D1 Three events A, B and C have $\mathrm{P}(A) = 0.4$, $\mathrm{P}(B) = 0.35$ and $\mathrm{P}(C) = 0.3$. $\mathrm{P}(A \cap B) = 0.1$, the events A and C are mutually exclusive, and $\mathrm{P}(C \mid B) = 0.2$.

(a) Find $\mathrm{P}(B \cap C)$. [2 marks]

Answer .....

(b) Draw a Venn diagram to show the probabilities of A, B and C. [4 marks]

Answer .....

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(c) Find $\mathrm{P}(A \mid B \cup C)$. [2 marks]

Answer .....

(d) Find $\mathrm{P}(B \mid A' \cap C')$. [2 marks]

Answer .....

Total for D1: 10 marks

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D2

$\mathrm{P}(A) = 0.4$, $\mathrm{P}(B \mid A) = 0.3$ and $\mathrm{P}(B \mid A') = 0.3$. Ollie writes: "We are not told $\mathrm{P}(B)$, so we cannot tell whether A and B are independent."

Ollie is wrong.

(a) Find $\mathrm{P}(B)$. [3 marks]

Answer .....

(b) Show that A and B are independent. [2 marks]

Answer .....

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(c) Prove that, for any events with $0 < \mathrm{P}(A) < 1$, if $\mathrm{P}(B \mid A) = \mathrm{P}(B \mid A')$ then A and B are independent. [4 marks]

Total for D2: 9 marks

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No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $X \sim \mathrm{B}(n, 0.5)$, where n is a positive integer.

(a) Show that $\mathrm{P}(X = 1 \mid X \geqslant 1) = \dfrac{n}{2^n - 1}$. [3 marks]

Answer .....

(b) Given that $\mathrm{P}(X = 1 \mid X \geqslant 1) = \dfrac{5}{31}$, find n. [2 marks]

Answer .....

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(c) For this value of n, find $\mathrm{P}(X = 2 \mid X \leqslant 2)$. [3 marks]

Answer .....

Total for E1: 8 marks

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 With $p = 0.5$, every outcome of the n trials has probability 0.5^n. Write $\mathrm{P}(X = 1)$ and $\mathrm{P}(X \geqslant 1)$ in terms of n.

Hint 2 Multiply the top and bottom of the fraction by 2^n.

Hint 3 For (b), try small values of n; $\dfrac{n}{2^n - 1}$ gets smaller as n grows, so only one value works.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can use the addition formula to find any one of P(A), P(B), P(A ∩ B) and P(A ∪ B). ○ ○ ○
- I can use P(A ∩ B) = P(A | B) P(B) and find P(A | B) from P(B | A). ○ ○ ○
- I can find a conditional probability for a discrete or binomial random variable. ○ ○ ○
- I can prove simple results about independence using the formulae. ○ ○ ○

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