Tree diagrams and conditional probability

131 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-statistics-year-2-tree-diagrams-and-conditional-probability/

Date

A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks in the exam.

A1 Needed today [3 marks]

$\mathrm{P}(A) = 0.3$, $\mathrm{P}(B \mid A) = 0.4$ and $\mathrm{P}(B \mid A') = 0.6$. Find $\mathrm{P}(B)$.

Answer .....

A2 Needed today [3 marks]

A tin holds 4 toffees and 5 mints. Two sweets are taken at random without replacement. Find the probability that both are mints, and the probability that one of each is taken.

Answer .....

A3 Needed today [2 marks]

$\mathrm{P}(A \cap B) = 0.12$ and $\mathrm{P}(B) = 0.3$. Find $\mathrm{P}(A \mid B)$.

Answer .....

A4 Common slip [2 marks]

On a tree diagram, $\mathrm{P}(A \cap B) = 0.08$ and $\mathrm{P}(A' \cap B) = 0.18$. Find $\mathrm{P}(A \mid B)$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

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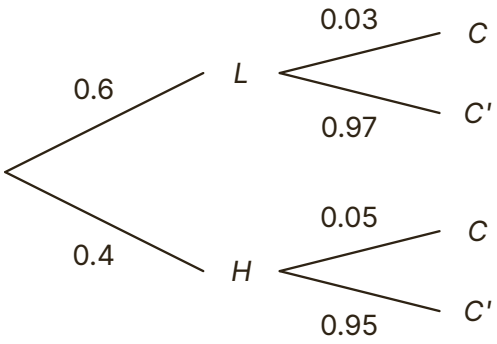
EXAMPLE 1

An insurer classes 60% of its drivers as low risk and the rest as high risk. In a year, 3% of low-risk drivers and 5% of high-risk drivers make a claim. A driver is chosen at random. (a) Find the probability that the driver makes a claim. (b) Given that the driver makes a claim, find the probability that the driver is high risk.



Hint The tree runs risk first, claim second. "Given a claim" runs it backwards: add up every path ending in C; that is the denominator.

B1 A garden centre buys 65% of its bulbs from supplier X and the rest from supplier Y. 12% of the bulbs from X and 20% of the bulbs from Y fail to flower. A bulb is chosen at random.

(a) Draw a tree diagram to show this information. [2 marks]

Answer .....

(b) Find the probability that the bulb fails to flower. [1 mark]

Answer .....

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(c) Given that the bulb fails to flower, find the probability that it came from X. [2 marks]

Answer .....

(d) Given that the bulb flowers, find the probability that it came from Y. [2 marks]

Answer .....

Total for B1: 7 marks

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EXAMPLE 2

A bag holds 3 black, 4 white and 2 grey counters. Two counters are taken at random without replacement. Given that the two counters are the same colour, find the probability that both are white.

Hint "Same colour" is three paths on the tree: black then black, white then white, grey then grey. Their total is the denominator.

B2 A pot holds 10 pens: 5 blue, 3 black and 2 red. Two pens are taken at random without replacement.

(a) Find the probability that the two pens are the same colour. [2 marks]

Answer .....

(b) Given that the pens are the same colour, find the probability that both are blue. [2 marks]

Answer .....

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(c) Given that the pens are different colours, find the probability that the first pen is black. [2 marks]

Answer .....

Total for B2: 6 marks

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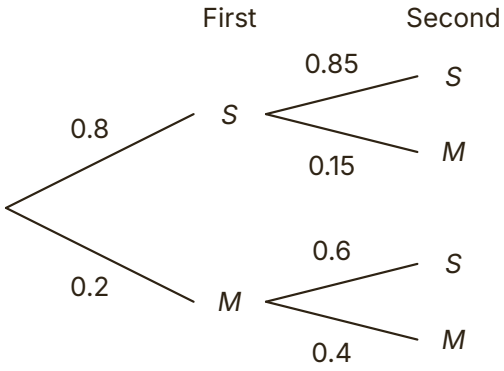
EXAMPLE 3

A player takes two penalties. The probability that she scores the first is 0.8. If she scores the first, the probability that she scores the second is 0.85; if she misses the first, it is 0.6. (a) Given that she scores at least one, find the probability that she scores both. (b) Given that she scores exactly one, find the probability that it was the first.



Hint "At least one" is every path except miss, miss. "Exactly one" is the two mixed paths. Use the complement for "at least one".

B3 In a quiz, the probability that Sam answers the first question correctly is 0.6. If he answers the first correctly, the probability that he answers the second correctly is 0.75; otherwise it is 0.4. Find the probability that

(a) he answers at least one correctly [2 marks]

Answer .....

(b) he answers both correctly, given that he answers at least one correctly [2 marks]

Answer .....

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(c) he answered the first correctly, given that he answered the second correctly. [2 marks]

Answer .....

Total for B3: 6 marks

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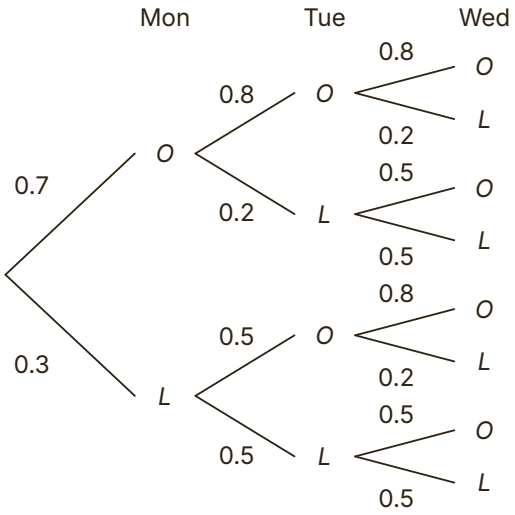
EXAMPLE 4

The probability that a bus is on time on Monday is 0.7. If it is on time one day, the probability it is on time the next day is 0.8; if it is late one day, the probability it is on time the next day is 0.5. (a) Find the probability that the bus is on time on Wednesday. (b) Given that the bus is late on Wednesday, find the probability that it was late on Monday.



Hint Each branch depends only on the day before. Multiply along a path; then add the paths that end the way you want.

B4 Leo plays one game of chess a day. The probability he wins on Monday is 0.5. If he wins a game, the probability he wins the next day is 0.7; if he does not win, it is 0.35.

(a) Find the probability that Leo wins on Wednesday. [3 marks]

Answer .....

(b) Given that Leo wins on Wednesday, find the probability that he won on Monday. [2 marks]

Answer .....

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(c) Given that Leo wins on both Monday and Wednesday, find the probability that he won on Tuesday. [2 marks]

Answer .....

Total for B4: 7 marks

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EXAMPLE 5

For two events, $\mathrm{P}(A) = 0.4$, $\mathrm{P}(B \mid A) = p$ and $\mathrm{P}(B \mid A') = 0.3$. Given that $\mathrm{P}(A \mid B) = 0.5$, find p.

Hint Write $\mathrm{P}(A \mid B)$ from the tree in terms of p, set it equal to 0.5 and solve.

B5 A bag holds 3 red counters and n green counters. Two counters are taken at random without replacement.

(a) Show that the probability that the first counter is green, given that the second is red, is $\dfrac{n}{n+2}$. [3 marks]

Answer .....

(b) This probability is $\dfrac{5}{7}$. Find n. [1 mark]

Answer .....

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(c) Find the probability that both counters are green, given that at least one is green. [2 marks]

C Practice

A-Level

C1 30% of the members of a club are seniors. 80% of seniors and 40% of the other members vote in an election. A member who voted is chosen at random. Ella writes: "The probability that this member is a senior is $0.3 \times 0.8 = 0.24$." Explain Ella's mistake and find the correct probability. [5 marks]

Answer .....

Total for B5: 6 marks

Answer .....

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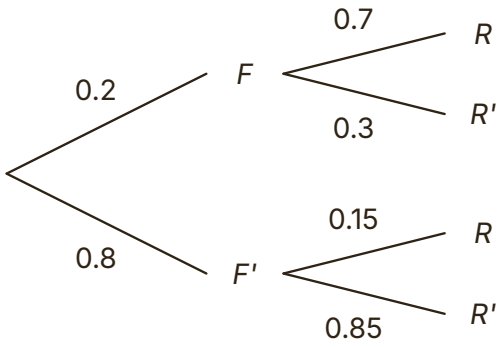
C2 A forecast says rain on 20% of days. When it says rain, it rains with probability 0.7; when it does not, it rains with probability 0.15. Max says: "It rained today, so the probability that rain was forecast is 0.7." Explain why Max is wrong, find the correct probability, and find the probability that rain was not forecast, given that it did not rain. [5 marks]



Answer .....

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C3 The probability that Joe is late for work on Monday is 0.2. If he is late on Monday, the probability he is late on Tuesday is 0.5; if he is not, it is 0.1. Joe works out the probability that he was late on Monday, given that he is late on Tuesday, as $\frac{0.1}{0.2} = 0.5$. Explain his mistake. Find the correct probability, and the probability that he was on time on Monday, given that he is on time on Tuesday. [5 marks]

Answer .....

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C4 A bag holds 3 red and 5 blue counters. Two are taken at random without replacement. Ola finds the probability that both are red, given that at least one is red, as $\dfrac{9/64}{39/64} = \dfrac{3}{13}$. Explain her mistake and find the correct probability. [5 marks]

Answer .....

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C5 Freya has up to three attempts to pass a test, and stops as soon as she passes. The probability that she passes at each attempt is 0.4, independently. Kai says: "Given that she passes, she is equally likely to pass at the first, second or third attempt, so each has probability $\frac{1}{3}$." Show that Kai is wrong by finding the probability that she passed at the first attempt, given that she passes. Find also the probability that she needed all three attempts, given that she passes. [6 marks]

Answer .....

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D1 A factory has three machines. Machine A makes 50% of the items, machine B makes 30% and machine C makes the rest. 2% of the items from A, 4% from B and 5% from C are faulty. An item is chosen at random.

(a) Draw a tree diagram to show this information. [2 marks]

Answer .....

(b) Find the probability that the item is faulty. [2 marks]

Answer .....

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(c) Given that the item is faulty, find the probability that it was made by machine B. [2 marks]

Answer .....

(d) Given that the item is not faulty, find the probability that it was made by machine A. [2 marks]

Answer .....

(e) The manager says: "Machine A makes half of the items, so half of the faulty items come from machine A." Comment on this. [2 marks]

Total for D1: 10 marks

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D2 A box holds 5 red and 3 blue counters. Three counters are taken at random, one after another, without replacement. Find the probability that all three are red, given that at least two are red. Find also the probability that the first two counters are both red, given that the third counter is red. [8 marks]

Answer .....

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D3 5% of a population have a condition. A test is positive for 90% of people with the condition and for 8% of people without it. A person is chosen at random and tested.

(a) Find the probability that the test is positive. [2 marks]

Answer .....

(b) Given that the test is positive, find the probability that the person has the condition, and explain why this is low. [3 marks]

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(c) A person who tests positive takes a second test, with the same probabilities and independent of the first given whether the person has the condition. Find the probability that a person who tests positive both times has the condition. [4 marks]

Answer .....

Total for D3: 9 marks

D4

> For two events, $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B \mid A) = 0.8$ and $\mathrm{P}(B \mid A') = 0.2$. Tia writes: "$\mathrm{P}(B) = 0.8 + 0.2 = 1$, so B is certain."

Tia is wrong.

(a) Explain Tia's mistake. [2 marks]

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(b) Find $\mathrm{P}(B)$. [2 marks]

Answer .....

(c) Find $\mathrm{P}(A \mid B)$. [2 marks]

Answer .....

(d) Find the probability that A happens, given that exactly one of A and B happens. [3 marks]

Answer .....

Total for D4: 9 marks

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D5 The probability that a football team scores first in a match is 0.6. If it scores first, the probabilities that it wins, draws and loses are 0.7, 0.2 and 0.1. If it does not score first, they are 0.25, 0.35 and 0.4.

(a) Find the probability that the team wins a match. [2 marks]

Answer .....

(b) Given that the team wins, find the probability that it scored first. [2 marks]

Answer .....

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(c) Given that the team does not lose, find the probability that it scored first. [3 marks]

Answer .....

(d) The results of two matches are independent. Given that the team wins at least one of the two, find the probability that it wins both. [3 marks]

Answer .....

Total for D5: 10 marks

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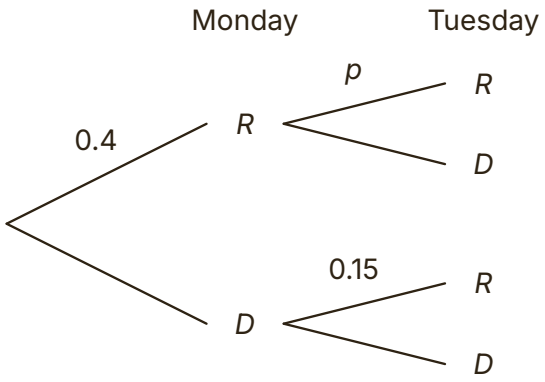
D6 The probability that it rains on Monday is 0.4. If it rains on one day, the probability it rains the next day is p; if it is dry, the probability it rains the next day is 0.15. The probability that it rains on Tuesday is 0.33.

(a) Find p, and the probability that it rained on Monday, given that it rains on Tuesday. [3 marks]



Answer .....

(b) Find the probability that Monday was dry, given that Tuesday is dry. [3 marks]

Answer .....

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(c) Find the probability that it rains on Wednesday. [3 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Bag A holds 3 red and 2 blue balls. Bag B holds n red and 4 blue balls. A ball is taken at random from bag A and put into bag B. A ball is then taken at random from bag B.

(a) Show that the probability that the moved ball was red, given that the ball taken from bag B is red, is $\dfrac{3n+3}{5n+3}$. [3 marks]

Answer .....

(b) This probability is $\dfrac{9}{14}$. Find n. [2 marks]

Answer .....

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(c) A second ball is taken from bag B, without replacement. Given that the first ball taken from bag B is red, find the probability that the second is also red. [3 marks]

Answer .....

Total for E1: 8 marks

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Draw a tree: the first stage is the colour moved, the second the colour taken from bag B, which now holds $n + 5$ balls.

Hint 2 $\mathrm{P}(\text{moved } R \mid \text{taken } R)$ is the red-red path over the two paths ending in red.

Hint 3 For (c), bag B has 10 balls after the move. Build the two-stage draw from it on each branch, then divide by $\mathrm{P}(\text{first taken from } B \text{ is red})$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can find P(first stage | second stage) from a tree diagram. ○ ○ ○
- I can find conditional probabilities such as given same colour, given at least one or given exactly one. ○ ○ ○
- I can use three-stage trees and condition on a later stage. ○ ○ ○
- I can find an unknown branch probability from a conditional probability. ○ ○ ○

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