Sum to infinity

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-sum-to-infinity/

Date

A Do Now

Quick questions to start. They use what you need today.

A1 From chapter 3 [2 marks]

A geometric series has first term 12 and common ratio $\frac{1}{3}$. Find S_4 as an exact fraction.

Answer .....

A2 Needed today [2 marks]

Solve $|2x - 1| < 1$.

Answer .....

A3 Common slip [2 marks]

Does the series $5 - 7.5 + 11.25 - \cdots$ have a sum to infinity? Give a reason.

Answer .....

A4 Needed today [2 marks]

Write $0.\dot{4}\dot{5}$ as a fraction in its simplest form.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the sum to infinity of $24 + 18 + 13.5 + \cdots$.

Hint Find r by dividing a term by the one before. Check $|r| < 1$ before using the formula.

B1 Find the sum to infinity of $50 - 20 + 8 - \cdots$. [4 marks]

Answer .....

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EXAMPLE 2

A geometric series has first term 12 and sum to infinity 48. Find the common ratio.

Hint Put the two numbers into $S_\infty = \dfrac{a}{1-r}$ and solve for r.

B2 A geometric series has first term 15 and sum to infinity 9. Find the common ratio and the third term. [4 marks]

Answer .....

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EXAMPLE 3

A geometric series has second term 8 and sum to infinity 36. Find the two possible values of the common ratio, and the first term in each case.

Hint Write $ar = 8$ and $\dfrac{a}{1-r} = 36$.

Replace a with $\dfrac{8}{r}$ to get a quadratic in r.

B3 A geometric series has second term 6 and sum to infinity 32. Find the two possible values of the common ratio, and the first term in each case. [6 marks]

Answer .....

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EXAMPLE 4

The series $1 + (2x - 1) + (2x - 1)^2 + \cdots$ has a sum to infinity. Find the range of values of x, and write the sum to infinity in terms of x.

Hint Here $r = 2x - 1$. The series converges when $|r| < 1$.

B4 [5 marks]

A geometric series has first term 2 and common ratio $\frac{x-3}{2}$. Find the range of values of x for which it has a sum to infinity, and find x when the sum to infinity is 8.

Answer .....

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EXAMPLE 5

A geometric series has first term 20 and common ratio 0.9. Find the least n for which $S_\infty - S_n < 1$.

Hint $S_\infty - S_n = \dfrac{ar^n}{1-r}$. Make 0.9^n the subject and take logs. Turn the inequality round when you divide by $\log 0.9$.

B5 A geometric series has first term 60 and common ratio 0.6. Find the least n for which $S_n > 149.9$. [5 marks]

Answer .....

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C1 Sam writes: "The sum to infinity of $6 + 9 + 13.5 + \cdots$ is $\dfrac{6}{1 - 1.5} = -12$." Explain why Sam is wrong. Then find the set of values of k for which the series $6 + 6k + 6k^2 + \cdots$ has a sum to infinity greater than 12. [5 marks]

Answer $\ldots\ldots < k < \ldots\ldots$

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C2 Use a geometric series to write $0.2\dot{3}\dot{6}$ as a fraction in its simplest form. [5 marks]

Answer .....

C3 A geometric series has first term 81 and common ratio $-\frac{1}{3}$. Find its sum to infinity, and the sum to infinity of its positive terms only. [5 marks]

Answer .....

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C4 A geometric series has first term 9 and third term 4. Find the two possible values of its sum to infinity. [4 marks]

Answer .....

C5 The sum to infinity of a geometric series is four times its first term. The third term is 18. Find the common ratio and the sum to infinity. [4 marks]

Answer .....

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D1 A geometric series has second term 12 and sum to infinity 75.

(a) Show that the common ratio r satisfies $25r^2 - 25r + 4 = 0$. [3 marks]

(b) Find the two possible values of r. [2 marks]

Answer .....

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(c) Given that $r > \frac{1}{2}$, find the least n for which $S_\infty - S_n < 0.5$. [4 marks]

Answer .....

Total for D1: 9 marks

D2

The series $5 + 5(2x - 3) + 5(2x - 3)^2 + \cdots$ has a sum to infinity. Leo writes: "It converges when $2x - 3 < 1$, so when $x < 2$."

Leo is wrong.

(a) Find the correct range of values of x. [3 marks]

Answer .....

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(b) Given that the sum to infinity is 4, find x. [3 marks]

Answer .....

(c) Explain why the sum to infinity cannot be 2. [2 marks]

Total for D2: 8 marks

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D3 The sum to infinity of a geometric series is $\frac{4}{3}$ times the sum of its first two terms. The third term is 6. Find the two possible values of the sum to infinity. [7 marks]

Answer .....

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D4 Consider the series $1 + \sin^2\theta + \sin^4\theta + \cdots$.

(a) Show that, where it exists, the sum to infinity is $\sec^2\theta$. [3 marks]

(b) Explain why there is no sum to infinity when $\theta = \dfrac{\pi}{2}$. [2 marks]

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(c) Find the values of θ in $0 < \theta < \pi$ for which the sum to infinity is 4. [5 marks]

Answer .....

Total for D4: 10 marks

D5 The rth term of a series is $u_r = 3 \times 2^{-r} + 4 \times 3^{-r}$, for $r = 1, 2, 3, \ldots$

(a) Show that the sum to infinity of the series is 5. [3 marks]

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(b) Show that $S_\infty - S_n = 3 \times 2^{-n} + 2 \times 3^{-n}$. [3 marks]

(c) Hence find the least value of n for which S_n is within 0.001 of the sum to infinity. [3 marks]

Answer .....

Total for D5: 9 marks

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D6 Recurring decimals can be written as geometric series.

(a) Use a geometric series to show that $0.\dot{9} = 1$. [3 marks]

(b) The digits p and q are whole numbers from 0 to 9. Prove that $0.\dot{p}\dot{q} = \dfrac{10p + q}{99}$. [4 marks]

Total for D6: 7 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A geometric series has sum to infinity 12. The series formed by squaring each of its terms has sum to infinity 48. Find the first term and the common ratio, and find the sum to infinity of the series formed by cubing each term. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 The squared series has first term a^2 and common ratio r^2.

Hint 2 Divide $\dfrac{a^2}{1-r^2} = 48$ by $\dfrac{a}{1-r} = 12$. Factorise $1 - r^2$.

Hint 3 You should get $\dfrac{a}{1+r} = 4$. The cubed series has ratio r^3.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can state the condition $|r| < 1$ and use it to find a range of values of x. ○ ○ ○

I can use $S_\infty = \dfrac{a}{1-r}$ forwards and backwards, including a negative ratio. ○ ○ ○

I can write a recurring decimal as a fraction using a geometric series. ○ ○ ○

I can find the least n for which S_n is within a given distance of S_∞. ○ ○ ○

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