

Surds and rationalising denominators

73 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-surds-and-rationalising-denominators/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Expand and simplify $(2x - 5)(2x + 5)$.
- A2** GCSE Higher [2 marks] Find the largest square factor of 180.

Answer

Answer

- A3** Last lesson [2 marks] Factorise $6x^2 + x - 15$.
- A4** From chapter 1 [2 marks] Evaluate $\left(\frac{243}{32}\right)^{-\frac{2}{5}}$.

Answer

Answer

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Simplify (a) $\sqrt{98}$ (b) $\sqrt{300} - \sqrt{75} + 2\sqrt{12}$.

Hint Find the largest square factor:
 $98 = 49 \times 2$.

B1 Simplify (a) $\sqrt{245}$ (b) $\sqrt{288} + \sqrt{50} - 3\sqrt{32}$. [3 marks]

Answer (a) (b)

EXAMPLE 2

Expand and simplify
 $(3 - \sqrt{5})(4 + 2\sqrt{5})$.

Hint Four products, as with $(x + a)(x + b)$.
 Then $\sqrt{5} \times \sqrt{5} = 5$.

B2 Expand and simplify (a) [4 marks]

$$\sqrt{6}(\sqrt{3} + \sqrt{8}) \quad (b) \quad (5 + 2\sqrt{3})^2.$$

Answer (a) (b)

EXAMPLE 3

Rationalise the denominator and
 simplify (a) $\frac{14}{\sqrt{7}}$ (b) $\frac{6 - \sqrt{2}}{3 + \sqrt{2}}$.

Hint In (b) multiply top and bottom by
 $3 - \sqrt{2}$. The bottom becomes $9 - 2$.

B3 Rationalise the denominator and [4 marks]

simplify (a) $\frac{18}{\sqrt{6}}$ (b) $\frac{5 + \sqrt{3}}{4 - \sqrt{3}}$, giving (b) in
 the form $p + q\sqrt{3}$.

Answer (a) (b)

EXAMPLE 4

Rationalise the denominator of

$$\frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}.$$

Hint Multiply top and bottom by $\sqrt{7} + \sqrt{3}$.
The bottom is $7 - 3$.

B4

Rationalise the denominator and

[3 marks]

simplify $\frac{\sqrt{11} - \sqrt{5}}{\sqrt{11} + \sqrt{5}}.$

Answer

EXAMPLE 5

Rationalise the denominator of

$$\frac{1}{(3 + \sqrt{5})^2}.$$

Hint Expand the bottom first:
 $(3 + \sqrt{5})^2 = 14 + 6\sqrt{5}$. Then multiply top and bottom by $14 - 6\sqrt{5}$.

B5

Rationalise the denominator and

[3 marks]

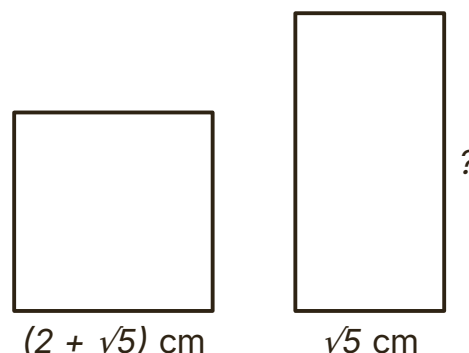
simplify $\frac{6}{(\sqrt{5} - 1)^2}.$

Answer

c Practice

A-Level

- C1** A square has sides of length $(2 + \sqrt{5})$ cm. A rectangle has the same area as the square, and one of its sides is $\sqrt{5}$ cm. Find the length of the other side of the rectangle, in the form $a + b\sqrt{5}$ where a and b are rational. [3 marks]



Answer cm

Not drawn accurately

- C2** Write $\frac{4 - \sqrt{2}}{3\sqrt{2} - 2}$ in the form $p + q\sqrt{2}$, where p and q are rational numbers. [4 marks]

Answer

- C3** Solve $5x - \sqrt{3} = x\sqrt{3} + 6$. Give your answer in the form $a + b\sqrt{3}$, where a and b are rational. [4 marks]

Answer

- C4** $p = 5 + 2\sqrt{6}$ and $q = 5 - 2\sqrt{6}$. Show that $\frac{1}{p} = q$, and find the value of $p^2 + q^2$. [4 marks]

Answer $p^2 + q^2 = \dots\dots$

D1

Ana is asked to write $\frac{10}{4 - \sqrt{6}}$ in the form $a + b\sqrt{6}$. She writes:

$$\frac{10}{4 - \sqrt{6}} \times \frac{4 + \sqrt{6}}{4 + \sqrt{6}} = \frac{40 + 10\sqrt{6}}{16 + 6} = \frac{20 + 5\sqrt{6}}{11}.$$

Ana has made a mistake.

(a) Explain the mistake Ana has made.

[1 mark]

(b) Write $\frac{10}{4 - \sqrt{6}}$ correctly in the form $a + b\sqrt{6}$.

[2 marks]

(c) Hence find the exact value of $\frac{10}{4 - \sqrt{6}} - \frac{10}{4 + \sqrt{6}}$.

[2 marks]

Answer

Answer

Total for D1: 5 marks

D2 Do not use a calculator in this question.

(a) Show that $(\sqrt{6} + \sqrt{2})^2 = 8 + 4\sqrt{3}$.

[2 marks]

- (b) Hence write $\sqrt{8 + 4\sqrt{3}}$ in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. [1 mark]

Answer

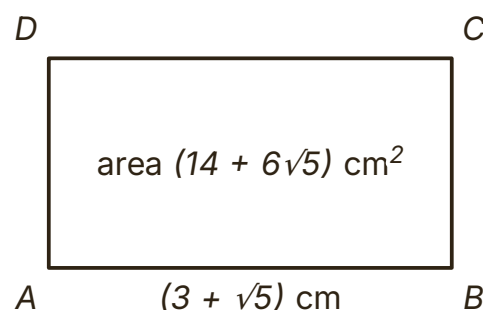
- (c) Hence write $\frac{4}{\sqrt{8 + 4\sqrt{3}}}$ in the form $\sqrt{c} - \sqrt{d}$, where c and d are integers. [3 marks]

Answer

Total for D2: 6 marks

- D3** The diagram shows a rectangle $ABCD$. $AB = (3 + \sqrt{5})$ cm and the area of the rectangle is $(14 + 6\sqrt{5})$ cm².

- (a) Show that $BC = (3 + \sqrt{5})$ cm. [3 marks]



Not drawn accurately

- (b) Hence find the perimeter of $ABCD$, in the form $a + b\sqrt{5}$. [1 mark]

Answer cm

(c) Show that $AC^2 = 28 + 12\sqrt{5}$.

[2 marks]

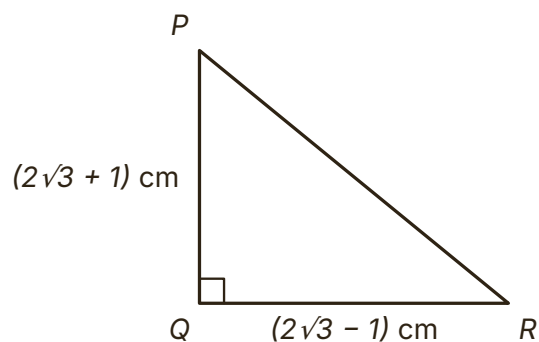
(d) A circle passes through A, B, C and D . Find the exact area of the circle, in the form $\pi(p + q\sqrt{5})$. [2 marks]

Answer cm^2

Total for D3: 8 marks

D4 In triangle PQR , angle $PQR = 90^\circ$, $PQ = (2\sqrt{3} + 1)$ cm and $QR = (2\sqrt{3} - 1)$ cm.

(a) Show that $PR = \sqrt{26}$ cm. [3 marks]



Not drawn accurately

(b) Find the exact area of triangle PQR . [2 marks]

Answer cm^2

- (c) The perpendicular from Q to PR has length h cm. Hence find h , [3 marks]
giving your answer in the form $\frac{a\sqrt{26}}{b}$ where a and b are integers.

Answer $h = \dots\dots$

Total for D4: 8 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The number $\sqrt{5} - \sqrt{2}$ is a root of an equation of the form $x^4 + px^2 + q = 0$, where p and q are integers. Find p and q . [6 marks]

Answer $p = \dots\dots$ $q = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Let $x = \sqrt{5} - \sqrt{2}$ and square it. One surd is left.
- Hint 2** $x^2 = 7 - 2\sqrt{10}$. Get the surd on its own:
 $x^2 - 7 = -2\sqrt{10}$. Square again.
- Hint 3** $(x^2 - 7)^2 = 40$, so $x^4 - 14x^2 + 49 = 40$ and
 $x^4 - 14x^2 + 9 = 0$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can simplify a surd and collect like surds. ☐ ☐ ☐

I can expand brackets that have surds in them. ☐ ☐ ☐

I can rationalise a denominator with one or two surds. ☐ ☐ ☐

I can give exact answers for lengths, areas and equations. ☐ ☐ ☐