

Tangents, chords and circles through triangles

74 marks



Hints online, answers free
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mathswariz.uk/worksheets/a-level-tangents-chords-and-circumcircles/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] **A2** Needed today [2 marks]
The line l passes through $(1, -2)$ and $(4, 4)$. Find the gradient of a line perpendicular to l .
Find the equation of the line through $(3, -1)$ with gradient $\frac{2}{3}$, in the form $ax + by + c = 0$ where a , b and c are integers.

Answer

Answer

- A3** Last lesson [2 marks] **A4** GCSE Higher [2 marks]
Show that the line $y = x + 4$ is a tangent to the circle $x^2 + y^2 = 8$, and find the point where it touches.
 P is $(1, 2)$, Q is $(3, 6)$ and R is $(7, 4)$. Work out PQ^2 , QR^2 and PR^2 . What do they tell you about angle PQR ?

Answer (.....,.....)

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The point $P(7, 5)$ lies on the circle $(x - 3)^2 + (y - 2)^2 = 25$. Find the equation of the tangent to the circle at P , in the form $ax + by + c = 0$.

Hint The tangent is perpendicular to the radius CP . Find the gradient of CP first.

B1 The circle C has [4 marks]

equation

$$x^2 + y^2 - 4x + 8y - 5 = 0.$$

Show that the point $P(5, 0)$ lies on C , and find the equation of the tangent to C at P in the form $ax + by + c = 0$.

Answer = 0

EXAMPLE 2

Find the equations of the two tangents to the circle $(x - 4)^2 + (y - 1)^2 = 18$ that have gradient 1.

Hint At each point of contact the radius is perpendicular to the tangent, so the radius has gradient -1 . Go out from the centre along a radius of gradient -1 .

B2 Find the equations of [5 marks]

the two tangents to the circle

$$(x + 2)^2 + (y - 3)^2 = 45$$

that have gradient $\frac{1}{2}$.

Answer $y = \dots\dots$ and $y = \dots\dots$

EXAMPLE 3

The triangle ABC has vertices $A(-3, 3)$, $B(5, 7)$ and $C(5, -1)$. Find the equation of the circle that passes through A , B and C .

Hint The centre is on the perpendicular bisector of AB AND on the perpendicular bisector of BC . Find where they cross.

B3 The triangle PQR has vertices $P(-4, 3)$, $Q(2, 5)$ and $R(8, -3)$. Find the equation of the circle that passes through P , Q and R . [6 marks]

Answer

c Practice

A-Level

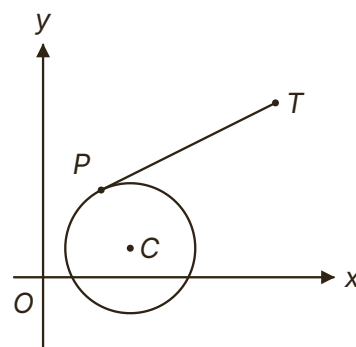
C1 The points $A(-3, 2)$, $B(1, 6)$ and $C(7, 0)$ lie on a circle. Show that AC is a diameter of the circle, and find the equation of the circle. [5 marks]

Answer

- C2** $A(4, 7)$ and $B(-2, -1)$ are the ends of a diameter of a circle. The point C lies on the positive x -axis and angle $ACB = 90^\circ$. Find the coordinates of C . [5 marks]

Answer (.....,.....)

- C3** The circle $x^2 + y^2 - 6x - 2y + 5 = 0$ has centre C . The point T is $(8, 6)$. A tangent from T touches the circle at P . Find the exact length of TP . [5 marks]



Not drawn accurately

Answer $TP = \dots\dots$

- D1** The points $A(1, -1)$, $B(-3, 1)$ and $C(5, q)$ lie on a circle, where q is a constant. Angle $BAC = 90^\circ$. [6 marks]

Find the value of q , and the equation of the circle.

Answer $q = \dots\dots$; $\dots\dots\dots$

- D2** The circle C has equation $(x + 1)^2 + (y - 4)^2 = 20$. The point $P(3, 6)$ lies on C . Sam finds the tangent to C at P . He writes:

The gradient from $(-1, 4)$ to $(3, 6)$ is $\frac{1}{2}$, so the tangent is $y - 6 = \frac{1}{2}(x - 3)$.

Sam is wrong.

- (a)** Explain the mistake Sam has made. [1 mark]

- (b)** Find the equation of the tangent to C at P . [3 marks]

Answer

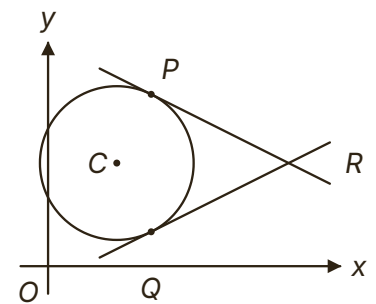
- (c) The tangent meets the x -axis at R . Find the coordinates of R . [4 marks]
Hence find the area of triangle CPR , where C is the centre of the circle.

Answer $R(\dots\dots, \dots\dots)$, area $\dots\dots$

Total for D2: 8 marks

- D3** The circle C has equation $x^2 + y^2 - 4x - 6y + 8 = 0$. The points $P(3, 5)$ and $Q(3, 1)$ lie on C . The tangents to C at P and at Q meet at the point R .

- (a) Find the centre and the radius of C . [2 marks]



Answer

Not drawn accurately

- (b) Find the equation of the tangent to C at P , in the form $ax + by + c = 0$. [3 marks]

Answer

- (c) The tangent at Q has equation $x - 2y - 1 = 0$. Find the coordinates of R . Hence find the area of the quadrilateral $CPRQ$, where C is the centre of the circle.

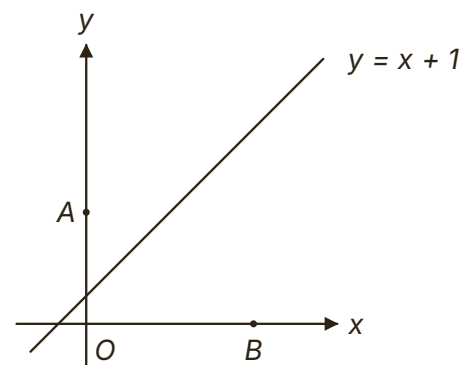
[4 marks]

Answer $R(\dots\dots, \dots\dots)$, area $\dots\dots$

Total for D3: 9 marks

- D4** A circle passes through the points $A(0, 4)$ and $B(6, 0)$. The centre of the circle lies on the line $y = x + 1$.

- (a) Find the equation of the perpendicular bisector of AB , in the form $ax + by + c = 0$. [3 marks]



Not drawn accurately

Answer

- (b) Hence find the equation of the circle. The circle meets the y -axis at A and at a second point D . Find the coordinates of D . [4 marks]

Answer ; $D(\dots\dots, \dots\dots)$

Total for D4: 7 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The point $T(7, 1)$ lies outside the circle $x^2 + y^2 = 25$. The two tangents from T touch the circle at P and Q . Find the coordinates of P and Q , and show that the two tangents are perpendicular to each other. [6 marks]

Answer (.....,.....) and (.....,.....)

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Angle OPT is 90° , so P lies on the circle that has OT as a diameter. So does Q .
- Hint 2** The circle with diameter OT is $x^2 + y^2 - 7x - y = 0$. Subtract it from $x^2 + y^2 = 25$ to get a straight line through P and Q .
- Hint 3** $7x + y = 25$, so $y = 25 - 7x$ and $x^2 - 7x + 12 = 0$: $P(3, 4)$ and $Q(4, -3)$. The tangents are $3x + 4y = 25$ and $4x - 3y = 25$, with gradients $-\frac{3}{4}$ and $\frac{4}{3}$, whose product is -1 .

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find a tangent to a circle using the fact that it is perpendicular to the radius. ○ ○ ○

I can use the perpendicular bisector of a chord to find the centre of a circle. ○ ○ ○

I can find the circle through three points, and spot when one side is a diameter. ○ ○ ○

I can work out lengths and areas made by tangents and radii. ○ ○ ○